Numerical Studies of the Linear and Non-linear Evolution of Density Waves Generated By Low Mass Planets in Protostellar Disk

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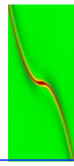


What?

We do hydro simulations to investigate the excitation and propagation of the density wave in gaseous protoplanetary disk driven by protoplanets, and to study the disk-planet interaction.

Why?

The primary goal is to verify the analytical theory by Goldreich & Tremaine 1980 and Goodman & Rafikov 2001, where the evolution of the density wave and disk-planet interaction in 2D case has been thoroughly studied in both linear and nonlinear stage. In addition, we investigate the effect of various numerical parameters in simulations. At last, we discuss a numerical problem in disk-planet simulations. Disk-planet interaction provides the torque for planet migration, drives the disk to evolve which leads to migration feedback, and shock formation introduced by nonlinear evolution may open gaps in the disk.

Why again? --- these simulations has been done numerous times!

Analytical theory tells us these quantities:

Primary: density profile in linear and nonlinear stage, torque density in physical space and in Fourier space, and where the shock happens (shocking length l_{sh}).

Secondary: total torque (integral of the torque density), and post-shock angular momentum decay.

Tertiary: migration rate (difference between torques from the inner and outer disk), and gap opening planet mass limit and process (accumulated effect of the post shock evolution).

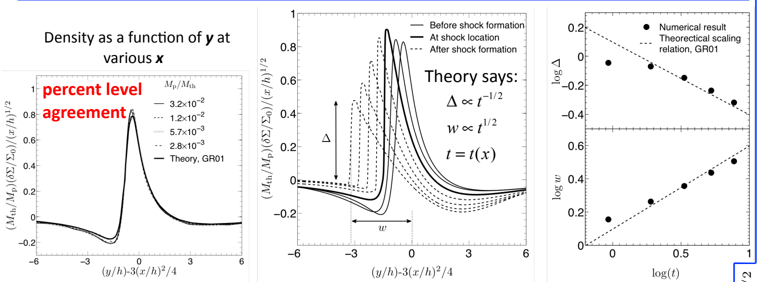
Vast majority of previous works focus on tertiary quantities, which are very derived quantities. We compare **each of the primary and secondary quantities** with the theory, at **percent level.** To achieve this goal, we use **extremely high spatial resolution (256 grid point per scale hight), accurate numerical algorithms, accurate planetary potential, and with low level of numerical viscosity** (α<1E-5)

How do we do this?

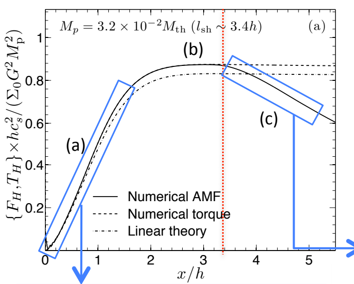
We use the new grid-based code Athena (Stone et al. 2008): Godunov scheme (good for shock capturing), fully conservative (mass, momentum, and energy), with very low level of numerical viscosity. Simulations are ***2D local shearing sheet inviscid*** simulations. The planet masses used are from ~0.003 to ~1 thermal mass ($M_{th}=c_s^3/G\Omega_p$), which corresponds to 4 lunar mass to 10 earth mass in MMSN.

The main results:

I. Density wave profile (azimuthal density cut at several radial positions)

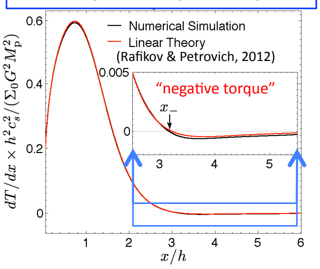


II. Angular momentum flux and torque calculation

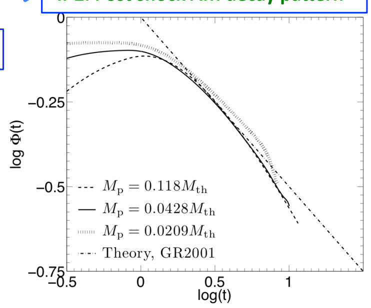


(a): Initial torque growth, as the wave collects Lindblad resonances.
(b): Reach a plateau after the torque excitation and before the shock. AMF agrees with the torque means angular momentum conservation.
(c): After the shock, angular momentum is no longer conserved, and it decays as the wave gets damped.

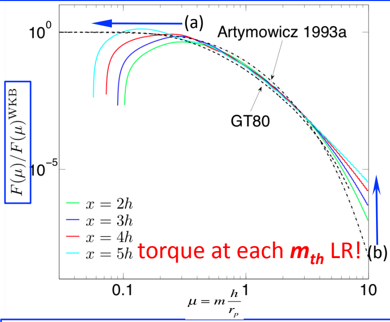
II-1. Torque density (the slope)

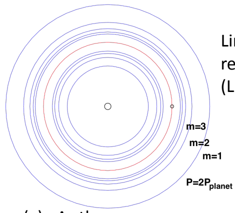


II-2. Post shock AM decay pattern



III. Torque density in Fourier space (at each m_{th} harmonics)



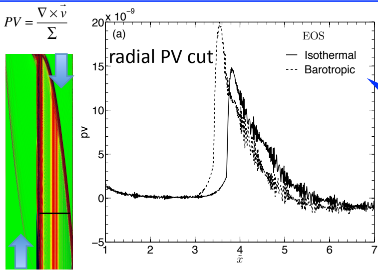


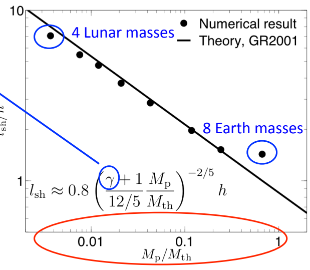
Lindblad resonance (LR) map

(a): As the wave moves away from the planet, it collects lower and lower LR.
(b): The nonlinear effect changes the power in higher m_{th} LR.

IV: Identify shocks (using potential vorticity PV), and confirm the l_{sh}-Mp relation

$$PV = \frac{\nabla \times \vec{v}}{\Sigma}$$



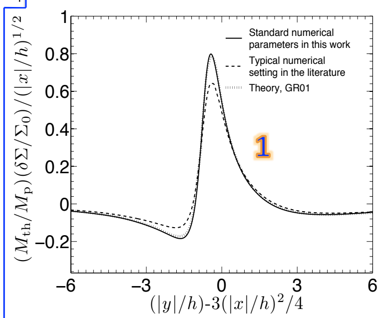


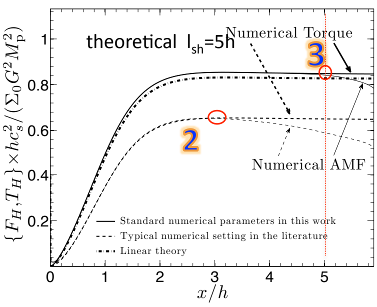
V: Effects of various numerical parameters

Low resolution, inaccurate numerical solver, and inaccurate planetary potential lead to:

1. Low density perturbation (~20%).
2. Shock (and dissipation) happens earlier.
3. Low final angular momentum of the wave (~25%).

These effects will have further consequences on planetary migration rate and gap opening process…





VI: A potential numerical problem.

When the dynamical time scale around the planet is not well resolved, the simulations would yield wrong results. In this case the wave develops a time dependent solution, and a spurious gap opening phenomenon emerges.