



# Planets and Disks

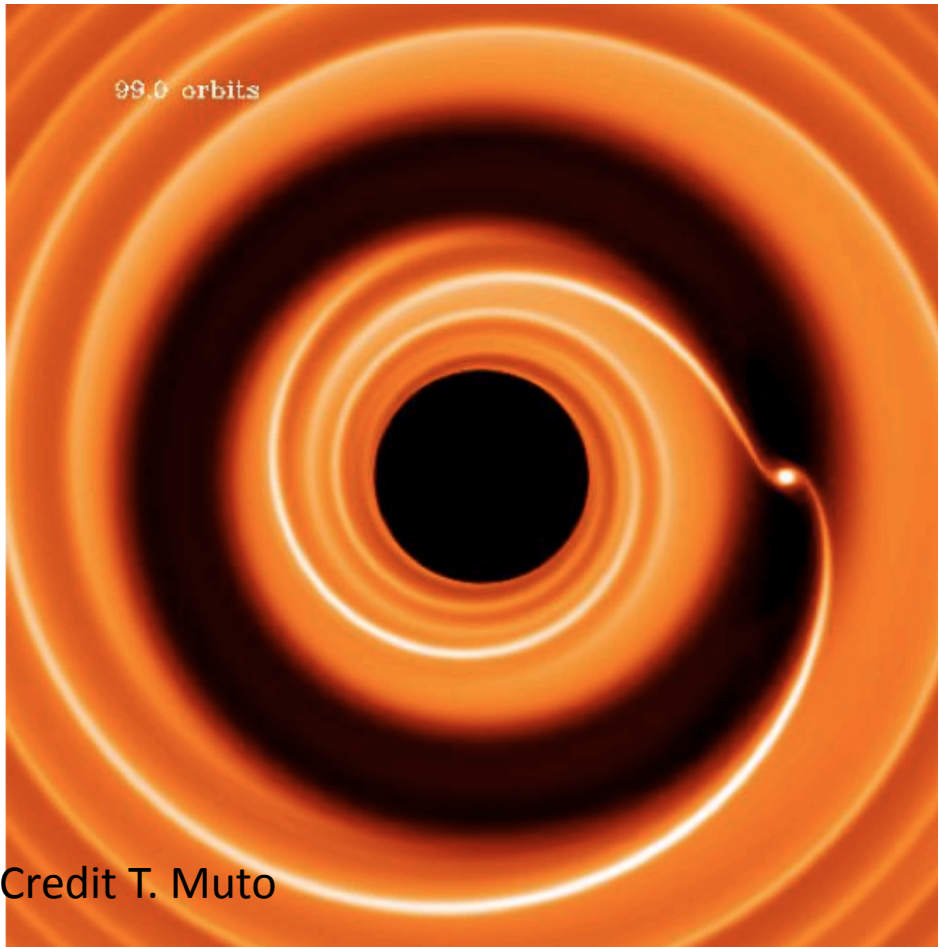
*Alice Quillen*

*University of Rochester*

# Theoretical overview

- Gaps, Clearings and sharp Edges in disks, how they are interpreted in terms of planets
- Planet/disk interactions and regimes
- Multiple planet systems and evolution

# Gap opening in a gas disk



Spiral density waves are driven at Lindblad resonances

Planet mass ratio  $q = M_p / M_*$

Torque on disk from each resonance depends on  $q$  and Fourier component of gravitational perturbation (Torque formula)

$$T_m = -f_m q^2 \Sigma r_p^4 \Omega_p^4$$

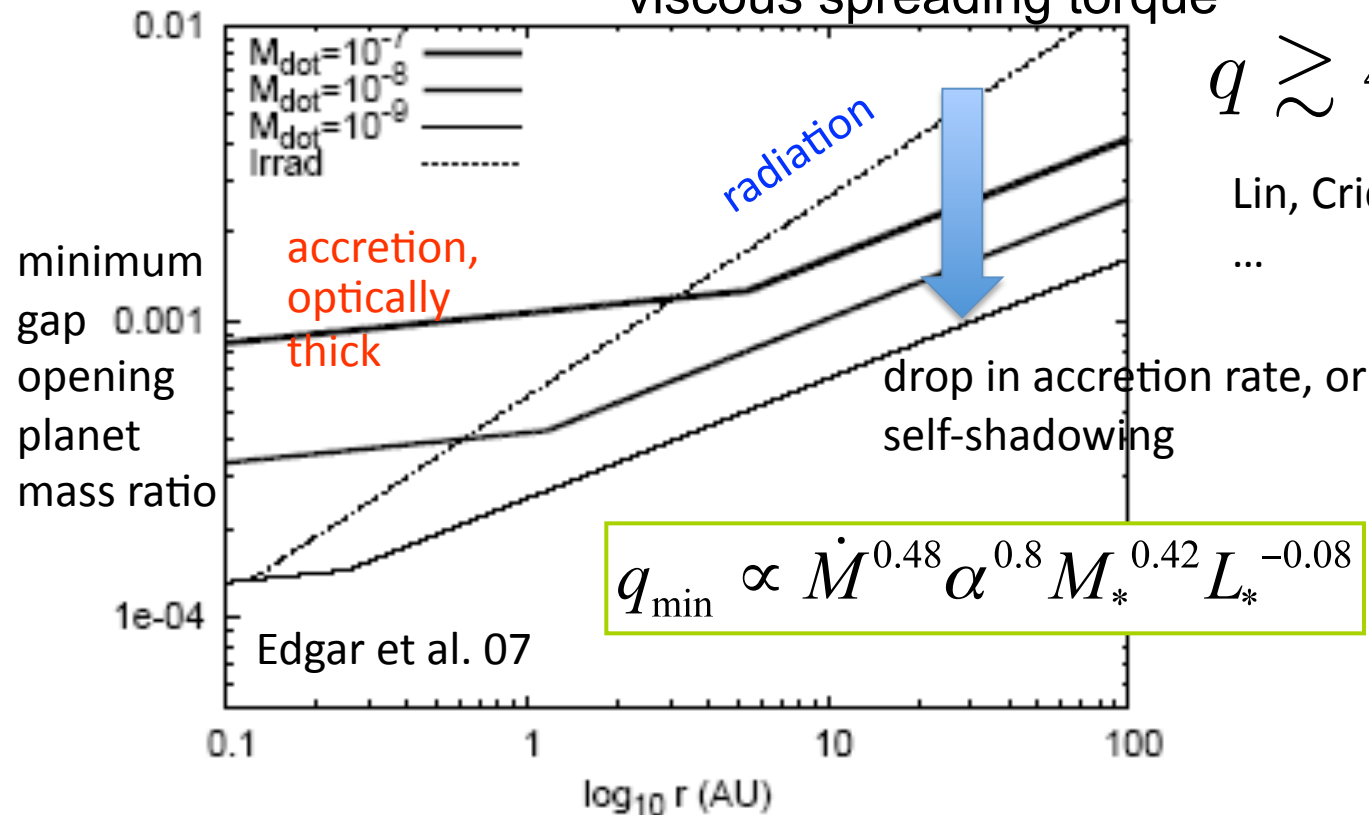
Torque independent of type of viscosity.

Stronger Lindblad resonances nearer the planet, but the location is shifted due to the sound speed → torque cutoff. Add the torques from resonances to estimate total

...

# Minimum Gap Opening Planet In an Accretion Disk

Assumption: Torques from waves launched by planet are not overcome by viscous spreading torque



$$q \gtrsim 40 \mathbf{Re}^{-1}$$

Lin, Crida, Bryden

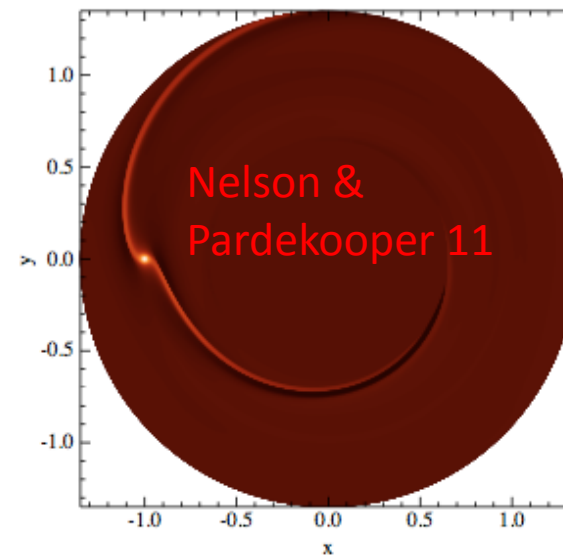
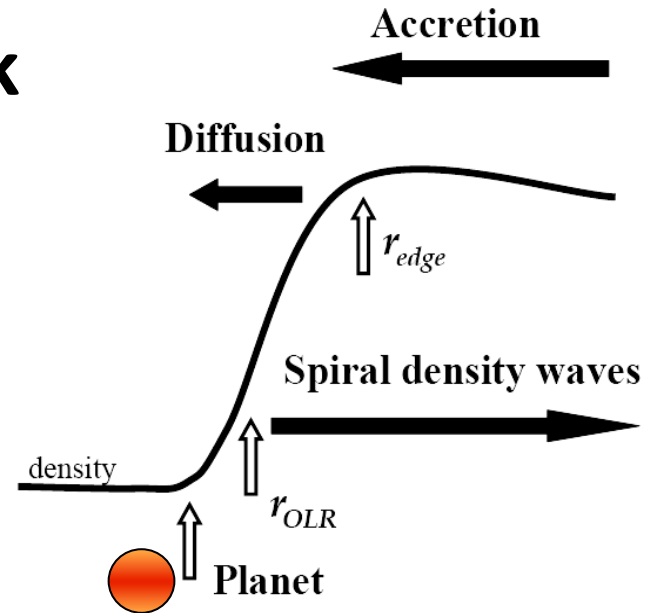
...

$$q_{\min} \propto \dot{M}^{0.48} \alpha^{0.8} M_*^{0.42} L_*^{-0.08}$$

For dense disks, large planets are required to account for disk edges  
**Self-shadowed** regions allow lower mass objects to open gaps

# Connections between disk morphology and planet

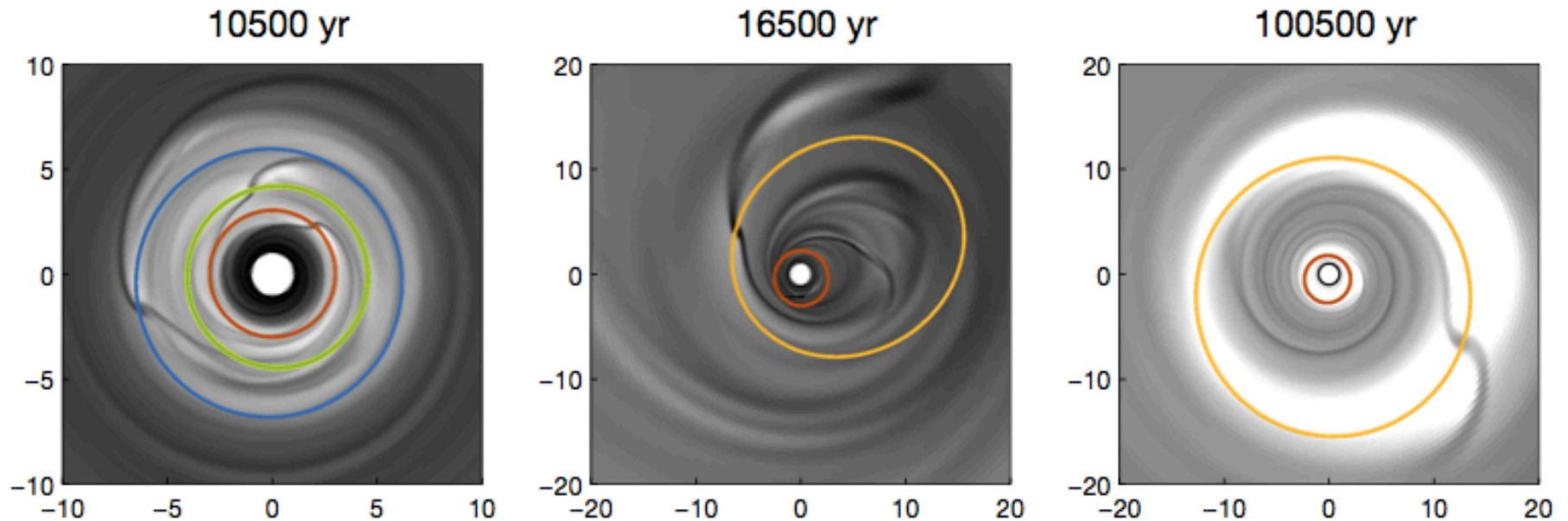
- Number of arms, slope, clearing time for gap, depend on planet mass.
- For small planets many waves are in phase  $\rightarrow$  single arm (Ogilvie)
- Higher planet masses leave a beat frequency between 2-3-4 armed waves



**Fig. 1** Surface density perturbation for a  $4 M_{\oplus}$  planet (located at  $x, y) = (-1, 0)$ ) embedded in a disc with  $h = 0.05$ , showing the prominent spiral wakes associated with Lindblad torques.

# Caveats/Complications

- Turbulence (e.g., Pardekooper, Ketchum)
- Multiple planet interactions
- Multiple planet scattering
- Resonance trapping
- Dead-zones
- Illumination
- Planet traps
- Planet formation+growth



Moekel & Armitage '11

# Gap opening in a dusty debris disk

- Taking the limit of low disk opacity  
Are spiral density waves driven at Lindblad Resonances?

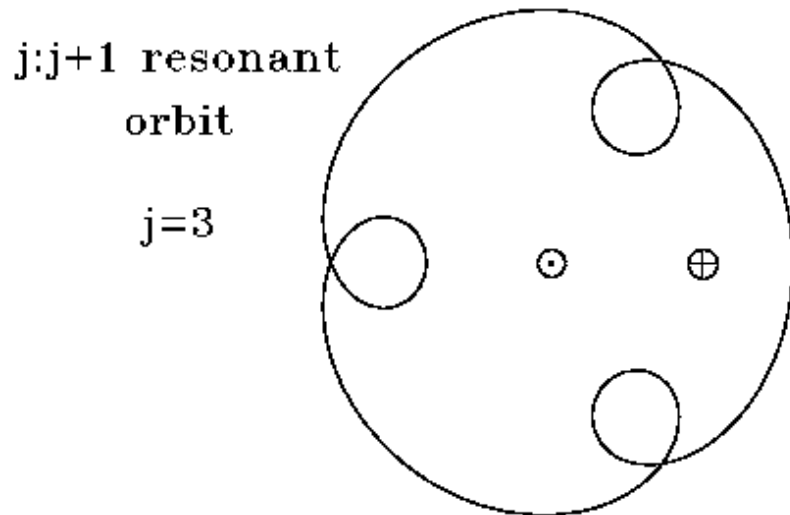
To answer this question first consider general properties of resonances

# Resonances

- Regions where small perturbations add up and no circular orbits exist
- When do perturbations become constructive?



# Resonant angle



In the frame rotating with the planet

$$jn_p - (j+1)n = 0$$

mean motions are integer multiples

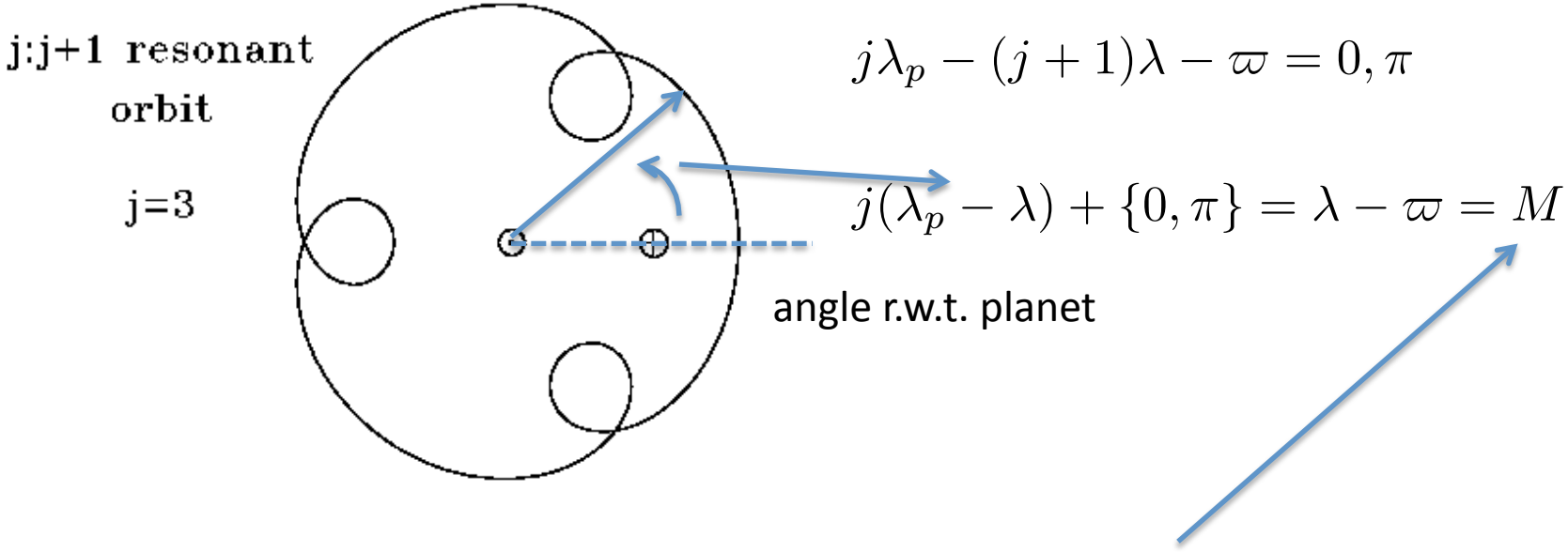
$$\phi = j\lambda_p - (j+1)\lambda = \text{constant like } 0 \text{ or } \pi$$

*Resonant angle* remains fixed

Librating resonant angle  $\leftrightarrow$  in resonance

Oscillating resonant angle  $\leftrightarrow$  outside resonance

# Resonant Angle



Mean anomaly which is zero at pericenter  
 Location in orbit radial oscillation related  
 to angle on sky w.r.t. planet

# Dimensional Analysis on the Pendulum

- $H$  units
- Action variable  $p$
- $a$
- $b$
- Drift rate  $db/dt$
- $\epsilon$

$$H(p, \theta) = a \frac{p^2}{2} + bp + \epsilon \cos \theta$$

$$\text{cm}^2 \text{ s}^{-2}$$

$$\text{cm}^2 \text{ s}^{-1}$$

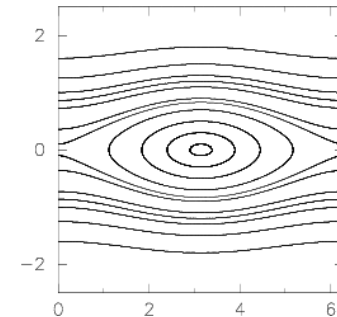
( $H=I\omega$ ) and  $\omega$  with 1/s

$$\text{cm}^{-2}$$

$$\text{s}^{-1}$$

$$\text{s}^{-2}$$

$$\text{cm}^2 \text{ s}^{-2}$$



Ignoring the distance from resonance we only have two parameters,  $a, \epsilon$

- Only one way to combine to get **momentum**
- Only one way to combine to get **time**

$$\sqrt{\epsilon/a}$$

$$1/\sqrt{a\epsilon}$$

# Resonant width and Libration period

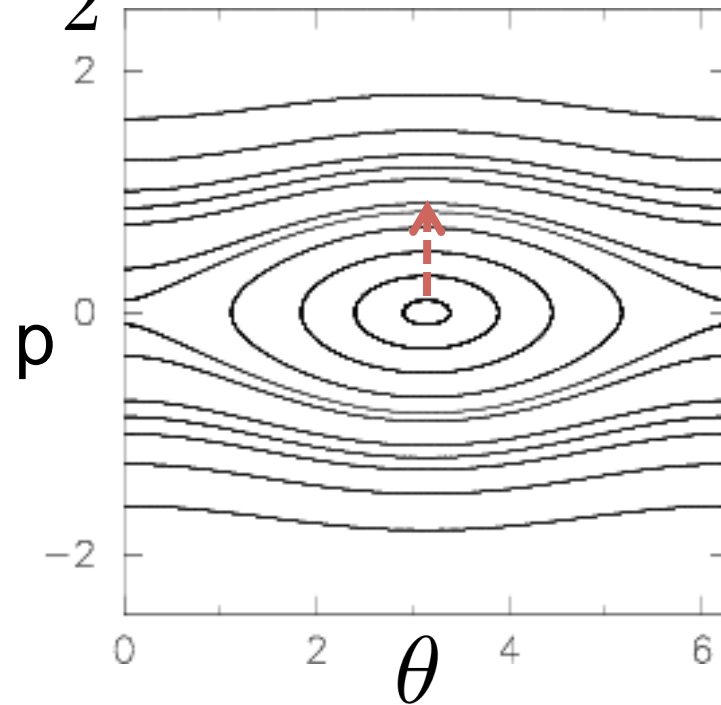
$$H(p, \theta) = a \frac{p^2}{2} + \epsilon \cos \theta$$

- **Resonant width** solve  $H(p, \theta) = 0$  for maximum  $p$ . Distance to separatrix

$$\Delta p = \sqrt{2\epsilon/a}$$

- **Libration timescale**, expand about fixed point  $\ddot{\theta} = -\epsilon a \theta$

Libration period  $\frac{2\pi}{\sqrt{a\epsilon}}$



note similarity between these expressions and those derived via dimensional analysis

# Dimensional analysis on the Andoyer Hamiltonian –

Low  $\epsilon$  expansion for mean motion or /and Lindblad resonances

$$H(p, \phi) = ap^2 + bp + \epsilon p^{k/2} \cos(k\phi)$$

- We only have two important parameters. ( $b$  sets if distance to resonance)

$a$  dimension  $\text{cm}^{-2}$

$\epsilon$  dimension  $\text{cm}^{2-k} \text{s}^{-2-k/2}$

- Only one way to form a **timescale** and one way to make a **momentum sizescale**.

$$\tau = (a\epsilon)^{2/3} \quad \text{for } k=1$$

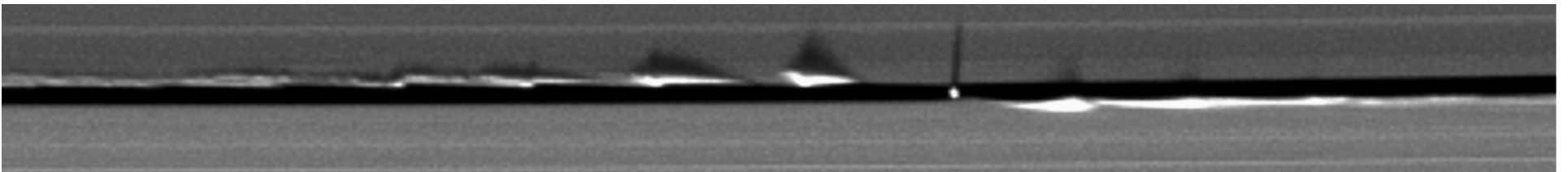
- To order of magnitude for  $j$ -th first order resonance  
 $\epsilon \sim \mu \delta^{-1} \exp(-j\delta) \quad a \sim j^2$

# Physical relevance

- $\tau^{-1}$  gives the libration timescale –relevant for
  - driving spiral density waves
  - proximity to resonance, sizes of kicks needed to push system in or out of resonance
- $\tau^{-2}$  rate of change of a frequency
  - The adiabatic limit (relevant for capture)
- The momentum sizescale –relevant for
  - sizes of motions in resonance
  - critical eccentricity ensuring capture in adiabatic limit
  - size of eccentricity jump if fail to capture and jump across the resonance

# Collision timescale and driving of spiral density waves

- Spiral density waves are not driven if the libration time of the Lindblad resonance is shorter than time between collisions.



Daphnis edge shadow

At low disk opacity spiral density waves are ineffective at pushing away a disk (e.g. Quillen 2005 in the context of the prediction of Fomalhaut B).

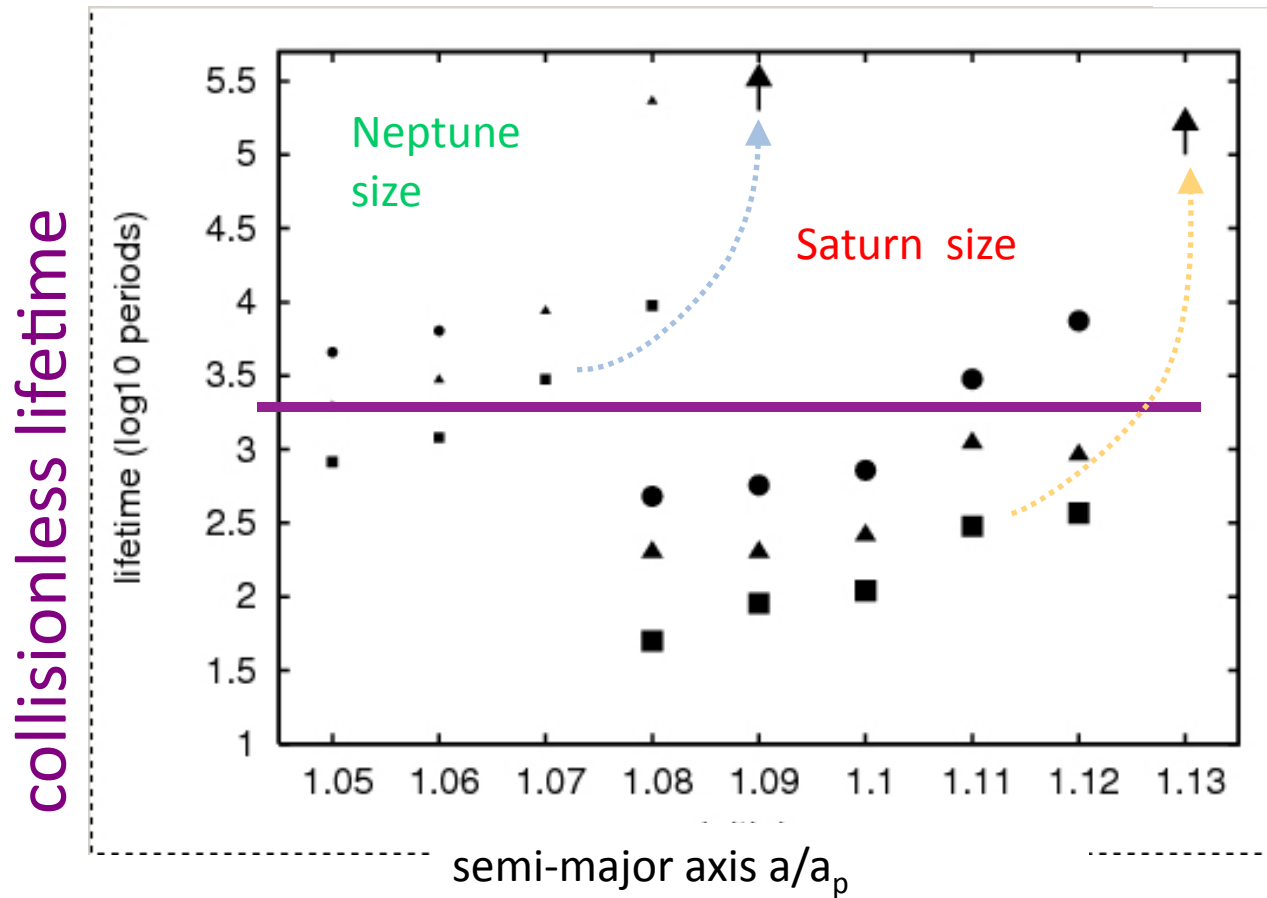
# Alternative boundaries for disk edges

- Mean motion resonances become stronger and denser closer to planet. They overlap at  $da \sim a \mu^{2/7}$
- $\mu^{2/7}$  law (Wisdom 1980) for the Width of chaotic zone
- Change in dynamics at this boundary
- The edge is potentially sharp.
- Proposed as a relevant boundary for the Fomalhaut disk edge (Quillen 2006)



# Chaotic zone boundary and removal time within

$$\frac{\partial}{\partial a} \left( D \frac{\partial N}{\partial a} \right) = \frac{N}{t_{removal}}$$



What mass planet will clear out objects inside the chaos zone fast enough that collisions will not fill it in?

$M_p > \text{Neptune}$



One the prediction of Fomalhaut B in 2006  
A single planet assumed to truncate both disk  
and account for eccentricity of dust belt

cleared out by  
perturbations from  
the planet

$M_p > \text{Neptune}$

nearly closed orbits  
due to collisions

eccentricity of ring  
equal to that of the  
planet

Assume that the edge of the ring is the  
boundary of the chaotic zone. Planet can't  
be too massive otherwise the edge of the  
ring would thicken or show structure →  
 $M_p < \text{Saturn}$

# Diffusive particle disk next to a planet

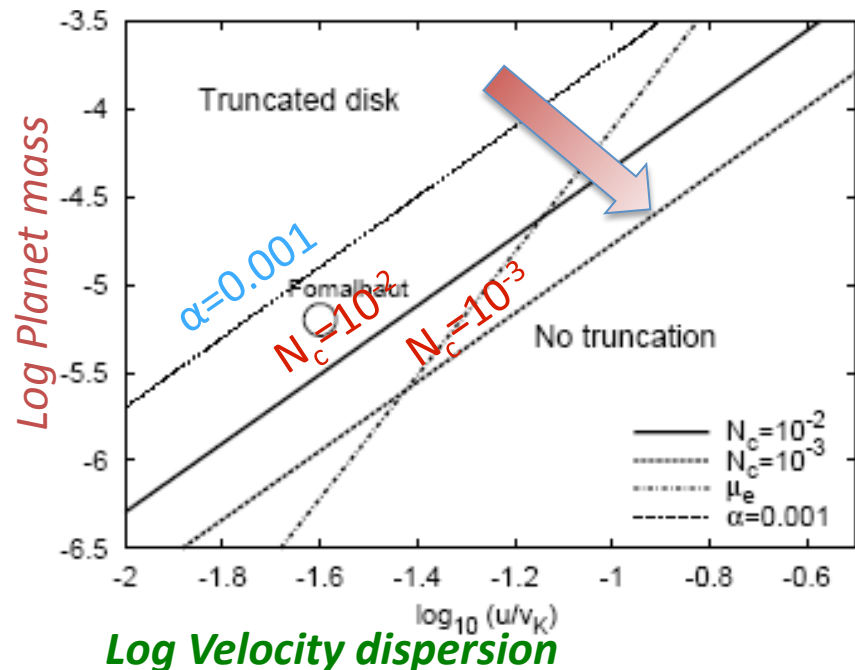
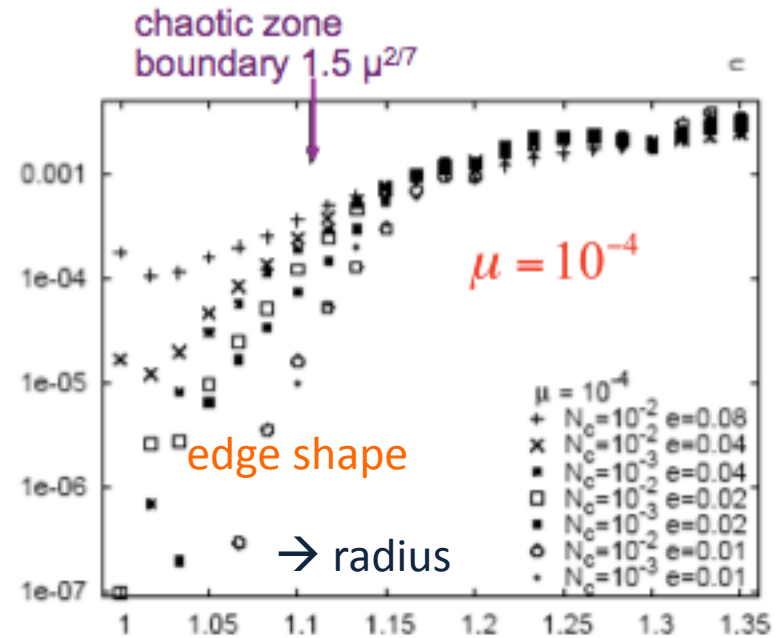
To truncate a disk a planet must have mass above

$$\log_{10} \mu > -6 + 0.43 \log_{10} \left( \frac{\tau_n}{5 \times 10^{-3}} \right) + 1.95 \left( \frac{u/v_K}{0.07} \right)$$

opacity

dispersion

low mass planets can open gaps in cold diffusive disks with long collision timescales



# Can the boundary be further away?

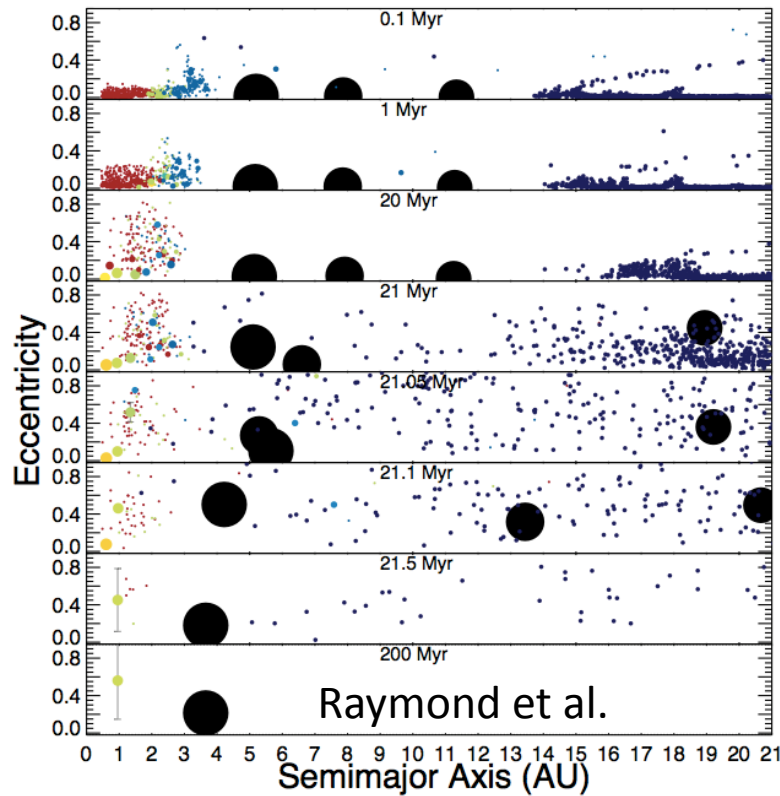
Why isn't Fom B exactly where it was predicted to be?

- Larger mass planet (Chiang et al)
- Excursions due to secular oscillations?
- Planet-planet scattering (e.g., Raymond, Moromartin)
- Planet eccentricity may not affect chaotic zone boundary (Quillen & Faber 07) though particle eccentricity and pericenter distributions do (Mustill & Wyatt 11)

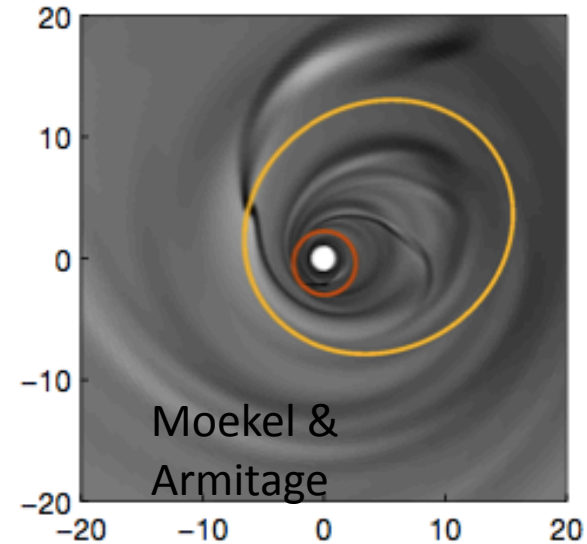
When can a massive planet be present  
in a disk and there is no gap?

# When can a massive planet be present in a disk and there is no gap?

1) Planet-planet Scattering into a gas disk, on short timescales →



16500 yr

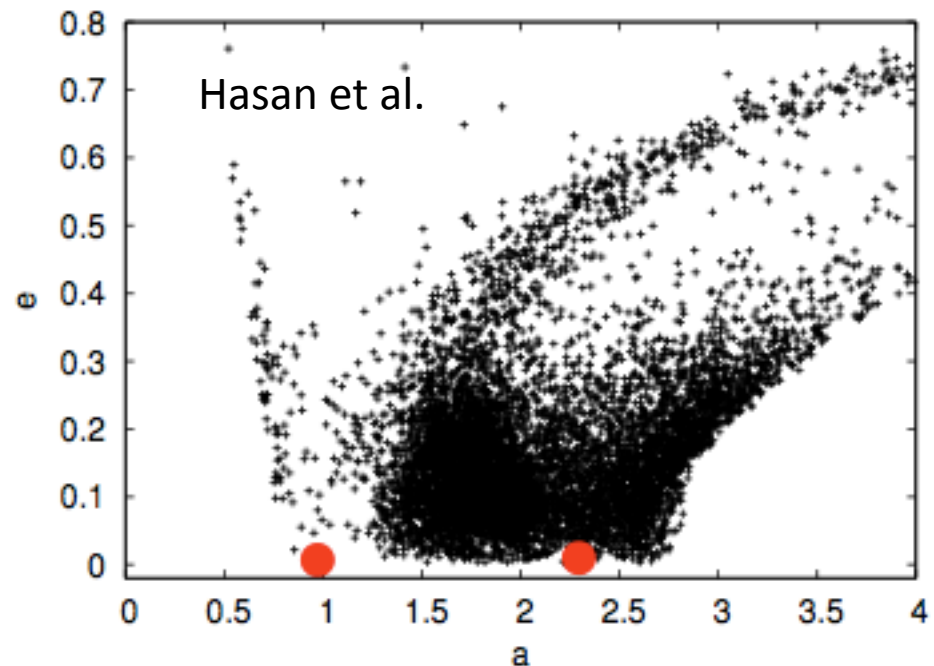


2) Planet-planet scattering into a debris disk, short timescales



# When can a massive planet be present in a disk and there is no gap?

3) Swift or runaway migration into a debris disk that is sufficiently massive to maintain migration (as discussed by Gomes, Duncan et al.



These settings can be classified.  
Either short timescale or  
The disk is dense compared to the planet mass  
→ Observable consequences such as dust production

When can you open a gap without a planet?



# When can you open a gap without a planet

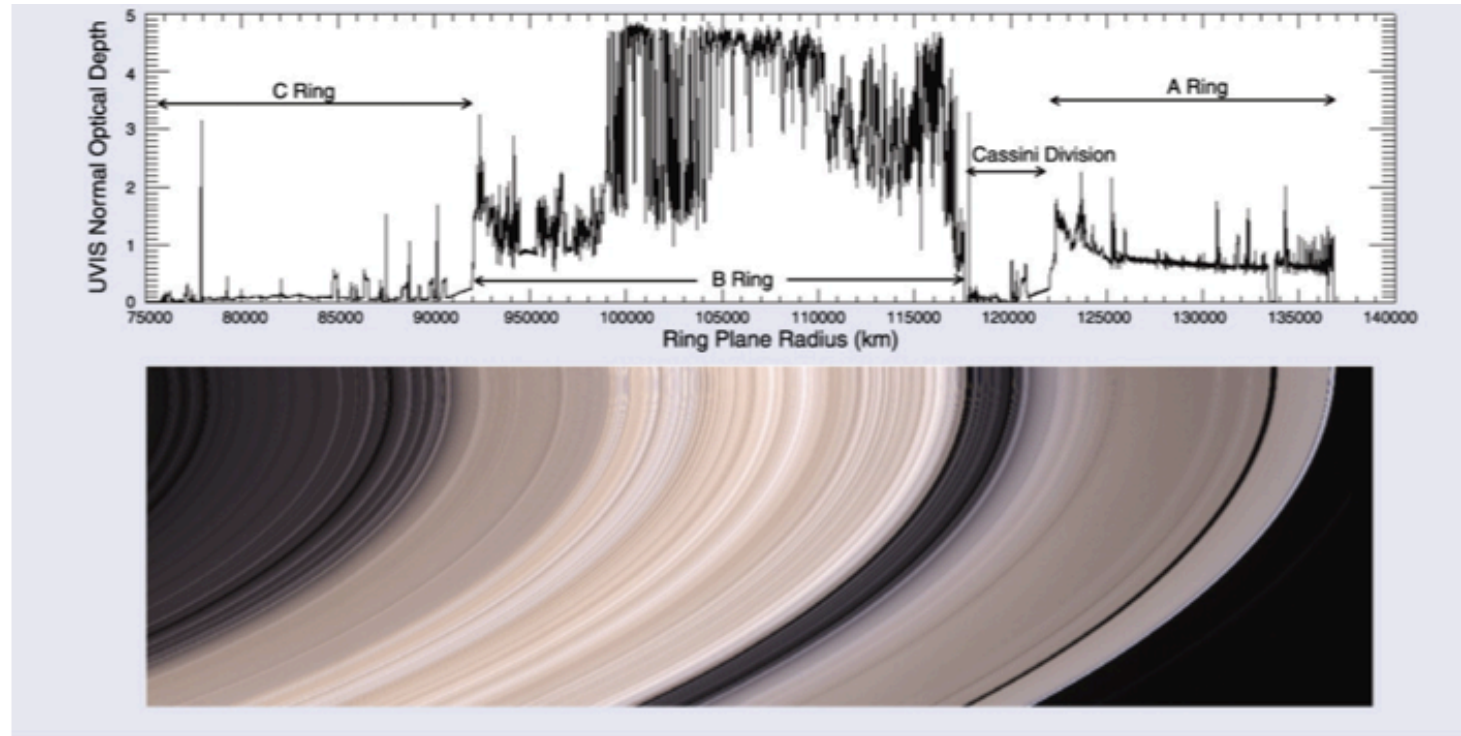
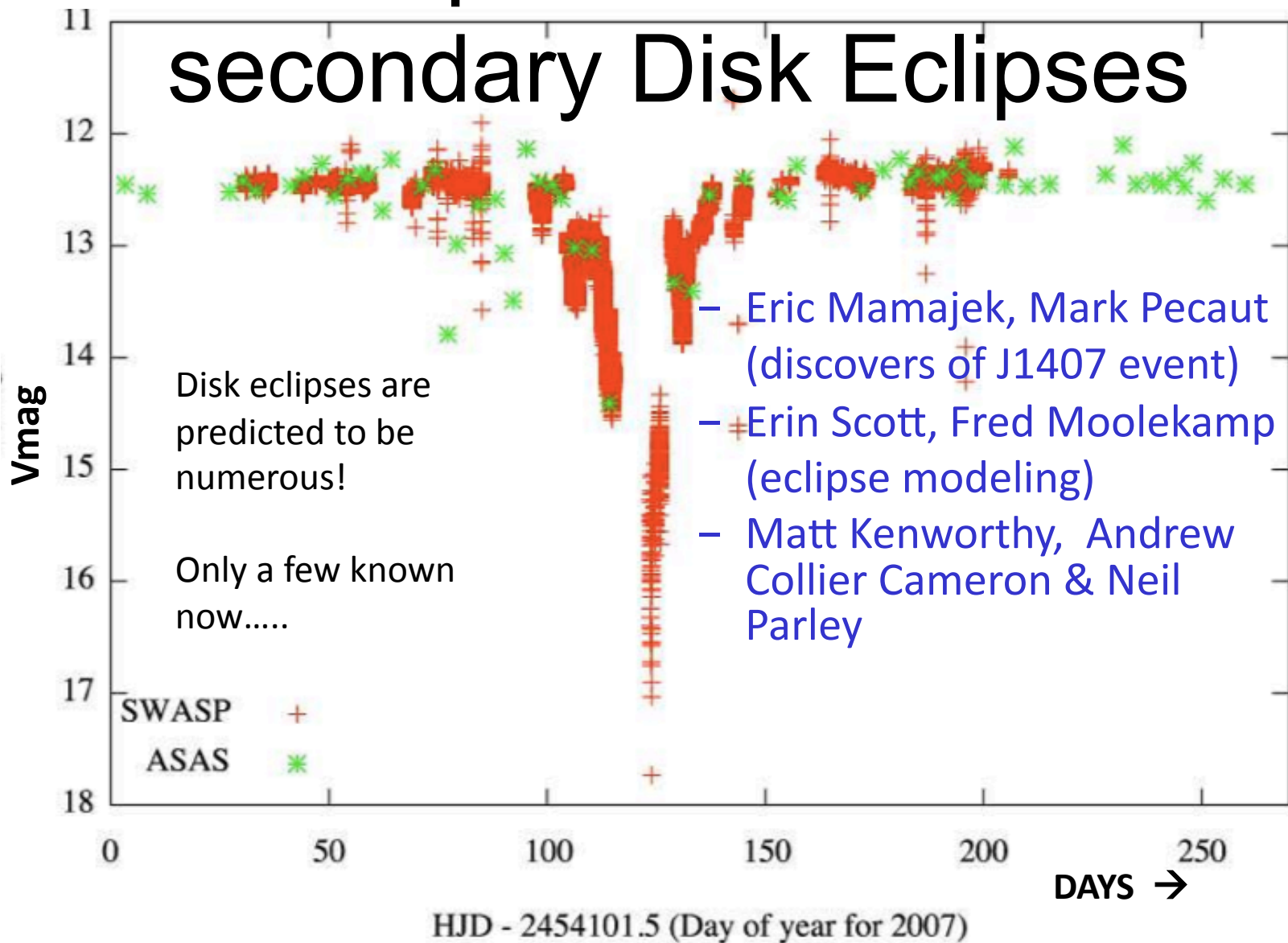


Fig. 11.— An optical depth profile (top) and true-color image (bottom) of Saturn's main ring system. Figure from *Cuzzi et al.* (2010)

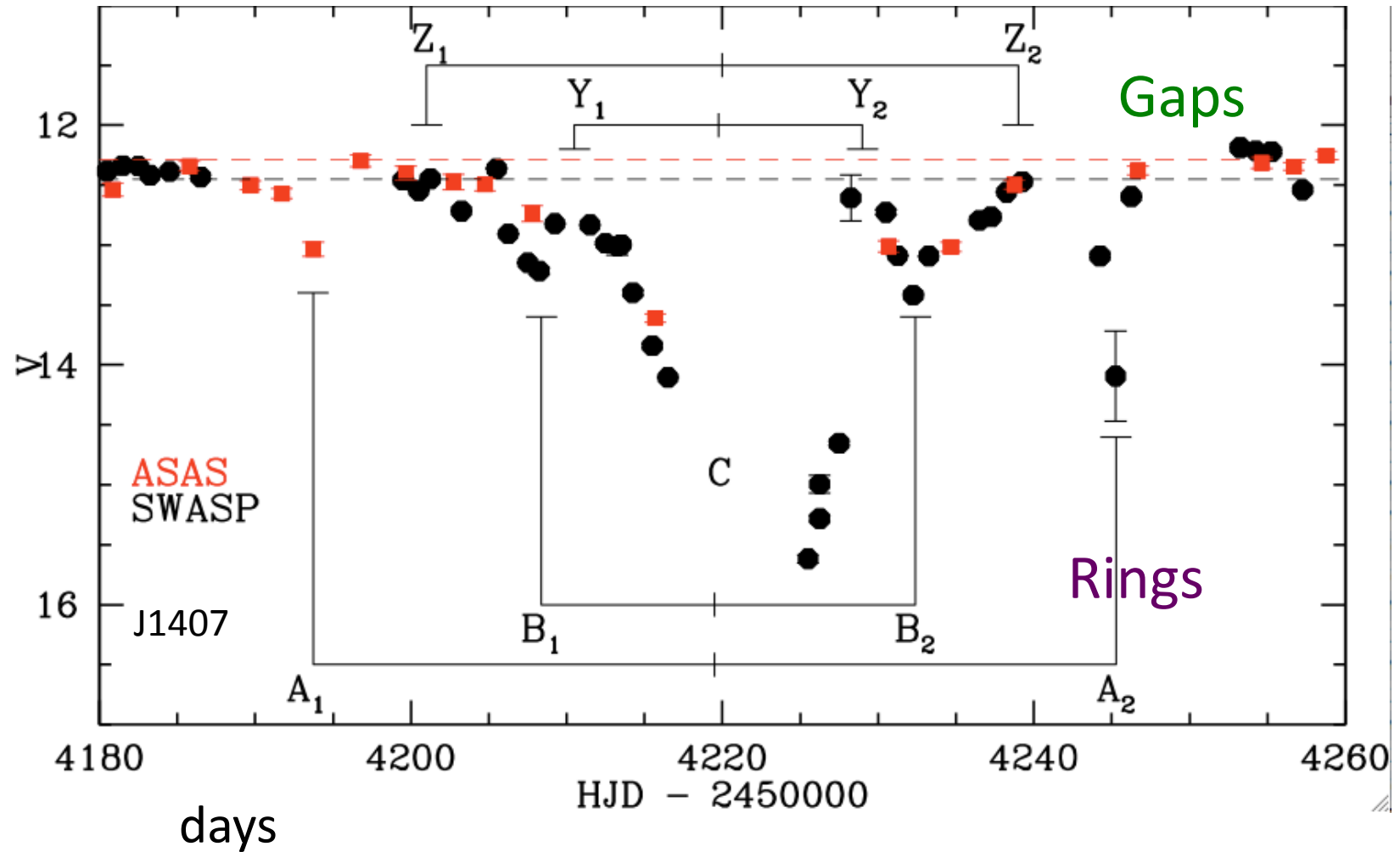
Required: cold or low velocity dispersion dense disks  
Note gaps are not necessarily empty, and depth is potentially observable.

Will we see exo-disks in a cold stage?

# Circum-planet and Circum-secondary Disk Eclipses

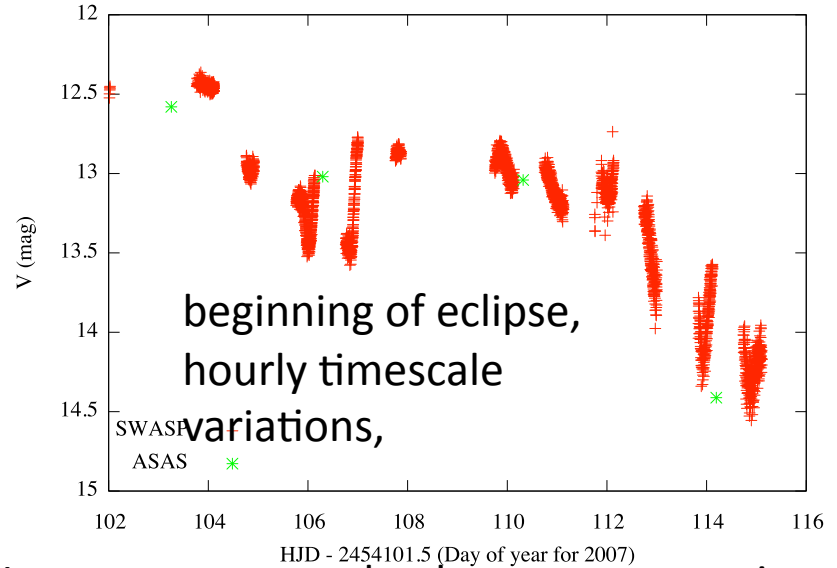
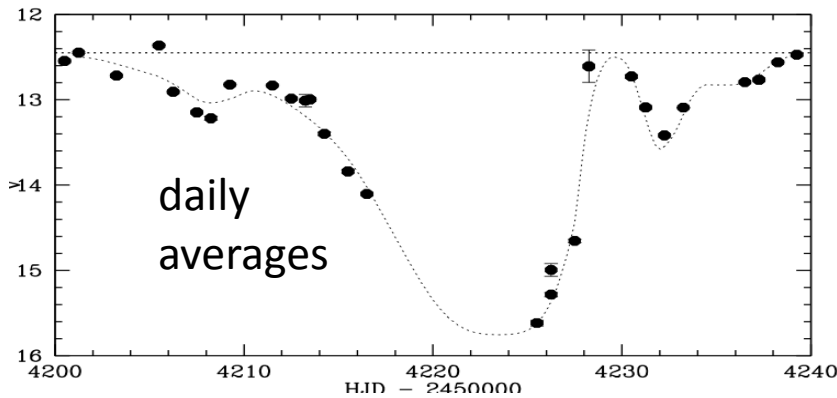


# J1407 Nightly averages



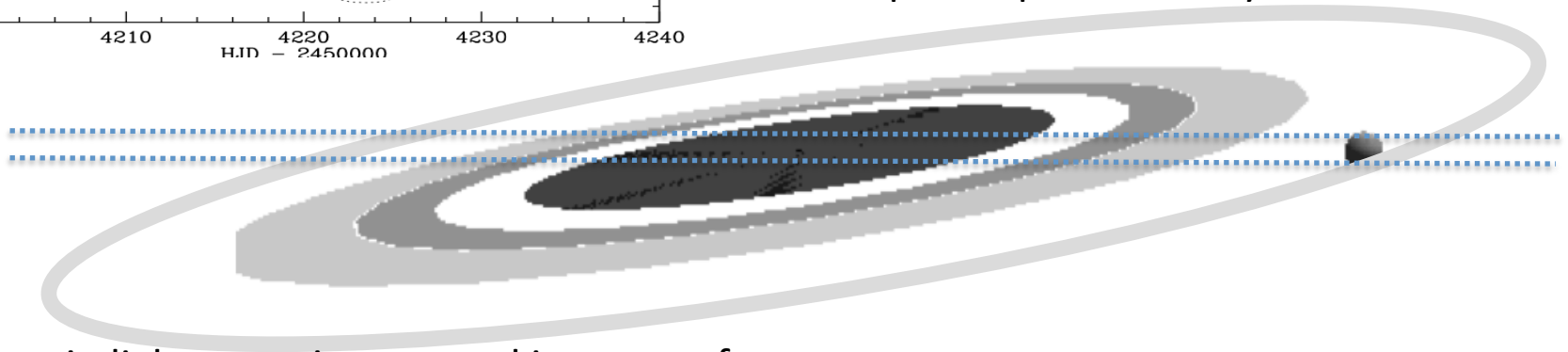
# Eclipse models

Fine structure in eclipse -> a very thin disk with lots of structure, like Saturn's rings  
 disk aspect ratio of  $h/r \sim 0.01$



Impact parameter leads to asymmetry in eclipse curve

Non unique eclipse model by Erin and Fred!



If gaps in light curve interpreted in terms of gap opening objects then their mass is of order  $10^{-3}$  of secondary

Gap opening objects likely pretty small

$$\frac{t_{gap}}{t_{eclipse}} \sim \left( \frac{m_{satellite}}{3m_2} \right)^{\frac{1}{3}}$$

# Forming planetary systems seen in eclipse

- J1407 star is 16Myr, solar type star.
- Forming planetary system, seen in occultation not reflected or thermal radiation
- potentially new views of forming planetary systems
- (+ eclipsing systems should be common)

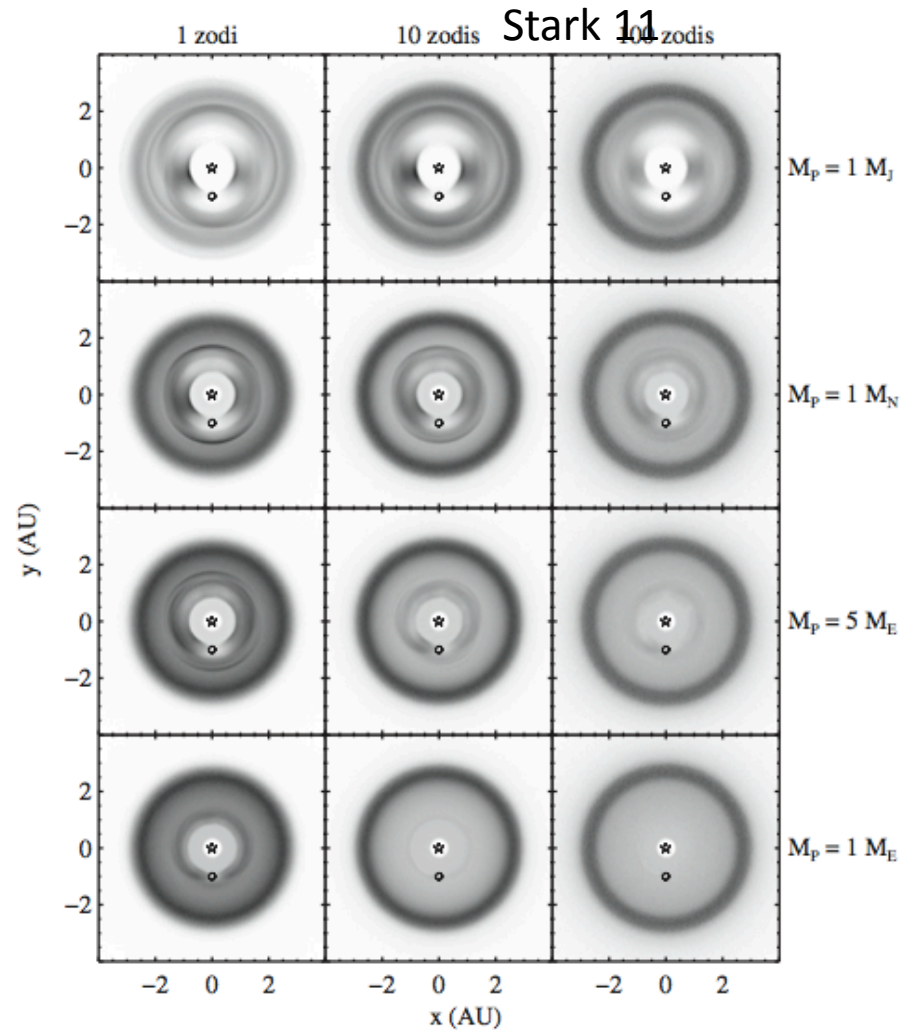
# Cold disk Components

May help explain

- Spirals and arcs (e.g. HD100546, HD141569, GM Aur, and other disks presented here)
- Clumps in some disks (e.g. AU Mic ...)
- Substructure in other debris disks

# When do you get interesting structures due to resonances?

- Resonance capture scenarios
- Liou & Zook, Ozerney et al., Kuchner & Holman early predictions of structure





# Another model

Fomalhaut

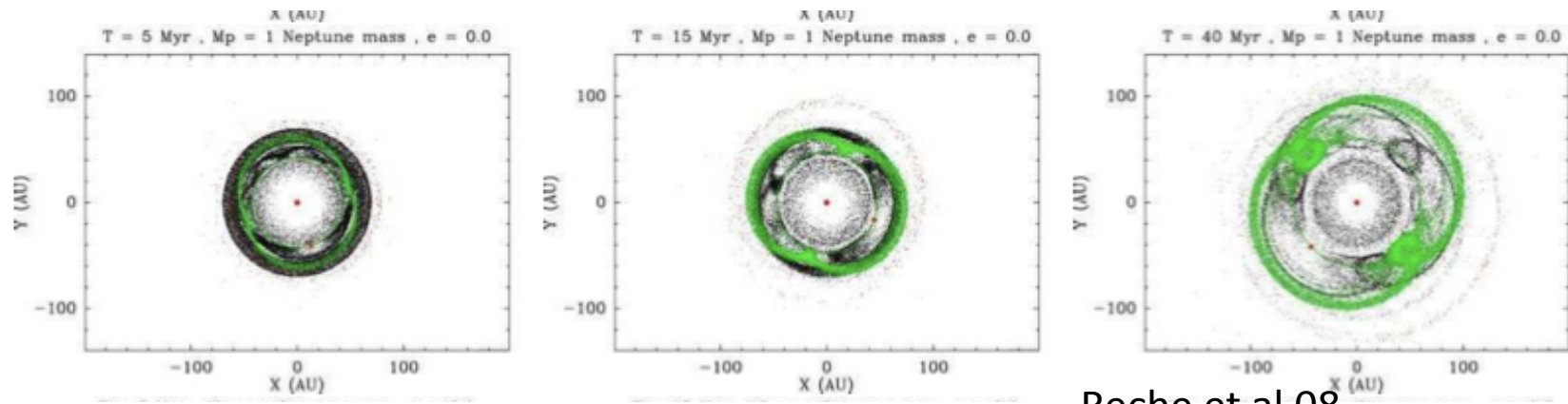


Top View

Perspective View

Adam Deller and Sarah Maddison's resonant capture model account for disk eccentricity but not sharp edge collisions ignored

# Structures generated by a migrating planet



Reche et al 08

low eccentricity

high eccentricity

4:3



3:2



2:1



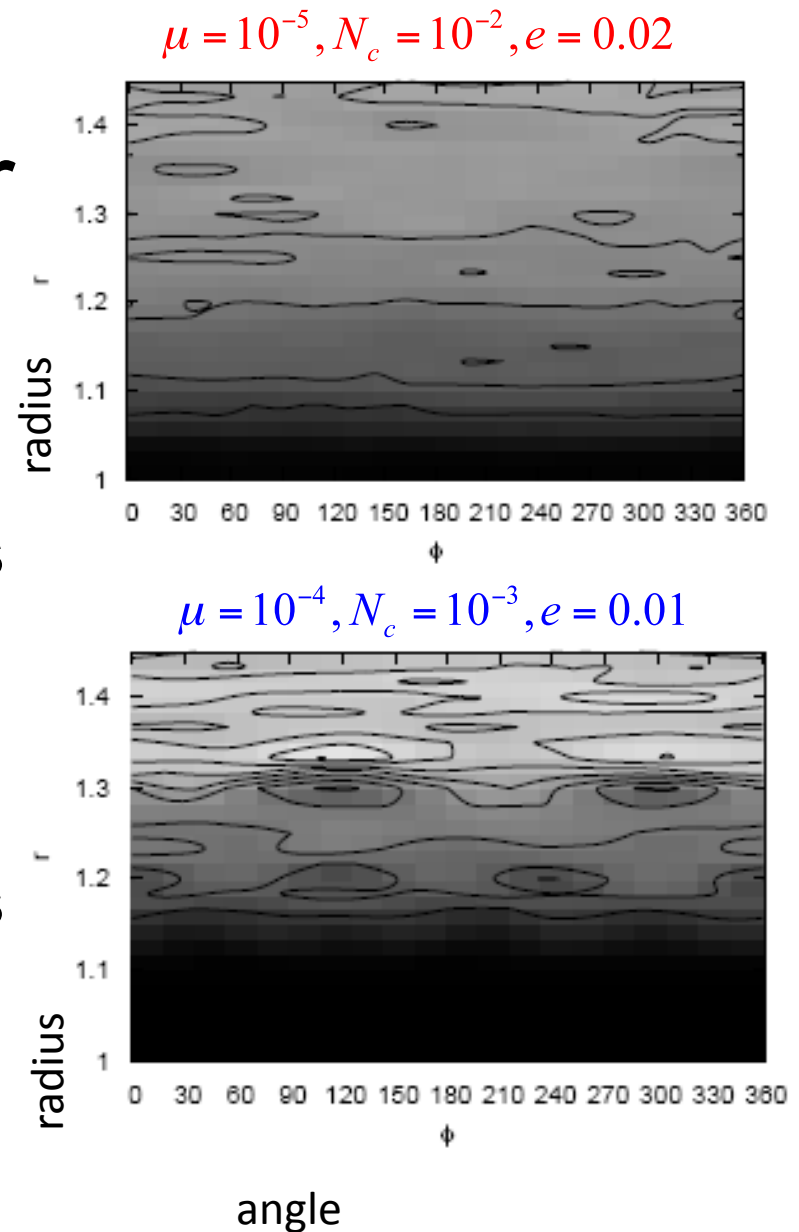
3:1



Kucher & Holman 03

# Morphology of diffusive disks near planets

- Featureless for low mass planets, high collision rates and velocity dispersions
- Particles removed at resonances in cold, diffuse disks near massive planets



# Predicting morphology

- Capture probability can be analytically predicted for all resonances either for dust spiraling via PR drag or due to migration of a planet. Identification of non-adiabatic limit, sensitivity to eccentricity and subresonances (Quillen 2006). Recent applications (Mustill & Wyatt 2011). Low eccentricity expansion is justified! (how many situations are we that lucky?)
- Using similar formalism one can estimate when turbulence or close encounters knock things out of resonance
- However: Resonant structure is smoothed by high eccentricity resonant population and estimating lifetimes in resonances is difficult.

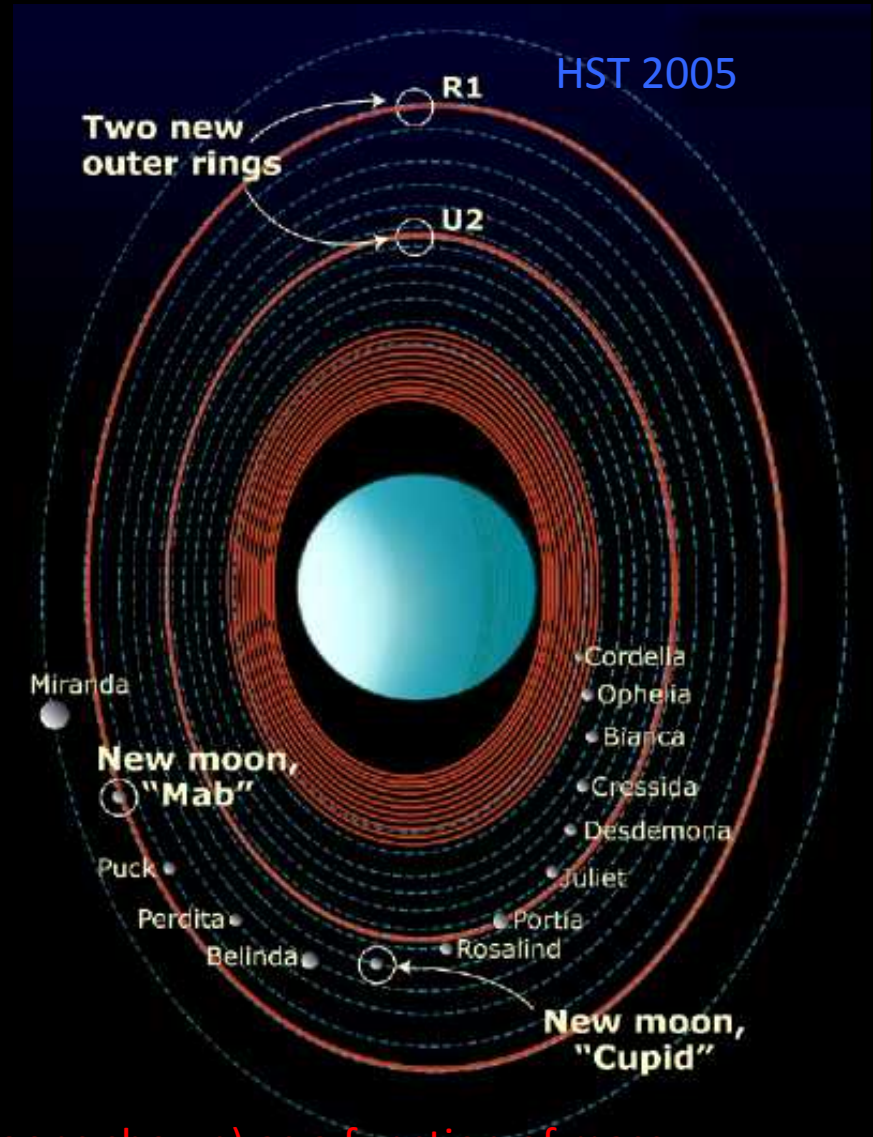
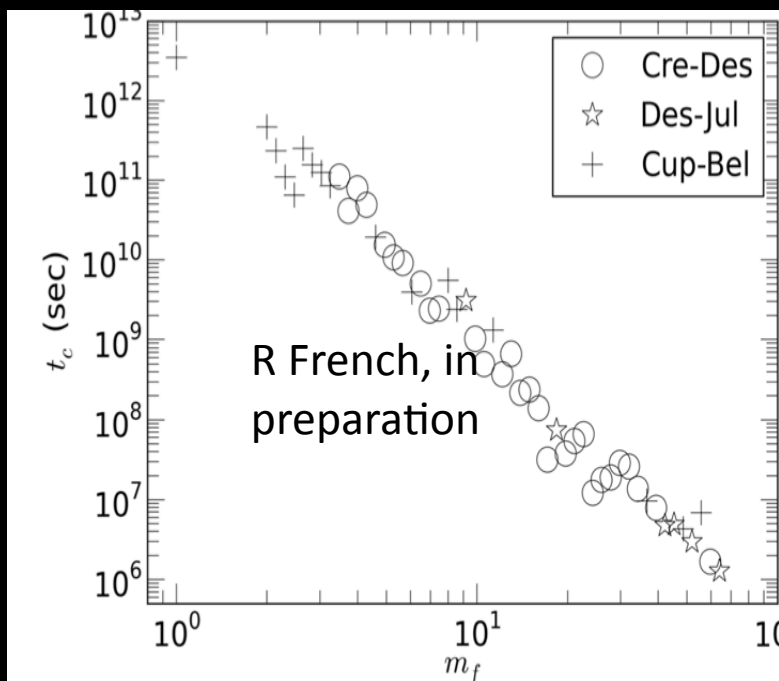
How many planets are needed to  
account for clearings?

# How many planets are needed to account for clearings?

- 0 - clearings are due to photoevaporation
- 1 – a single high eccentricity planet is all you need (+ Kozai resonance or previous scattering event)
- Many – Stability timescales leveraged to estimate the number of rocky/icy/gas giants in circular orbits needed to clear the system during its lifetime.

Note: clearings are not necessarily empty → observables  
hopefully differentiate between these possibilities

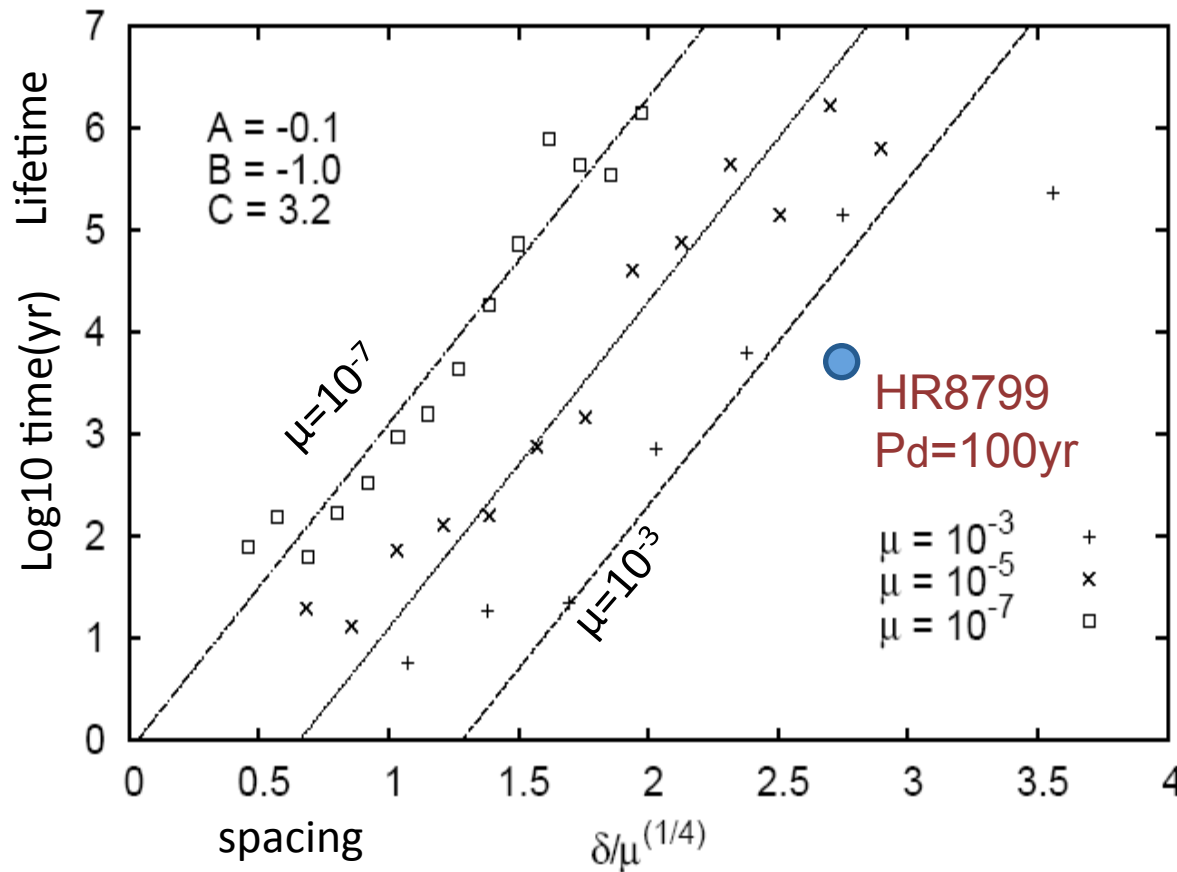
# Integrated Orbit Crossing timescales for the densely packed Uranian satellite system



Time of first orbit crossing (each pair of moons shown) as a function of mass scaling factor. Power-law relation first seen in integrations by Martin Duncan & Lissauer 97 and John Chambers et al. 96

# Disk Clearing by Planets

$$\log(t_e) = A + B \log(\mu/10^{-7}) + C(\delta/\mu^{1/4})$$



Relationship between spacing, clearing time and planet mass

Invert this to find the spacing, using age of star to set the stability time.

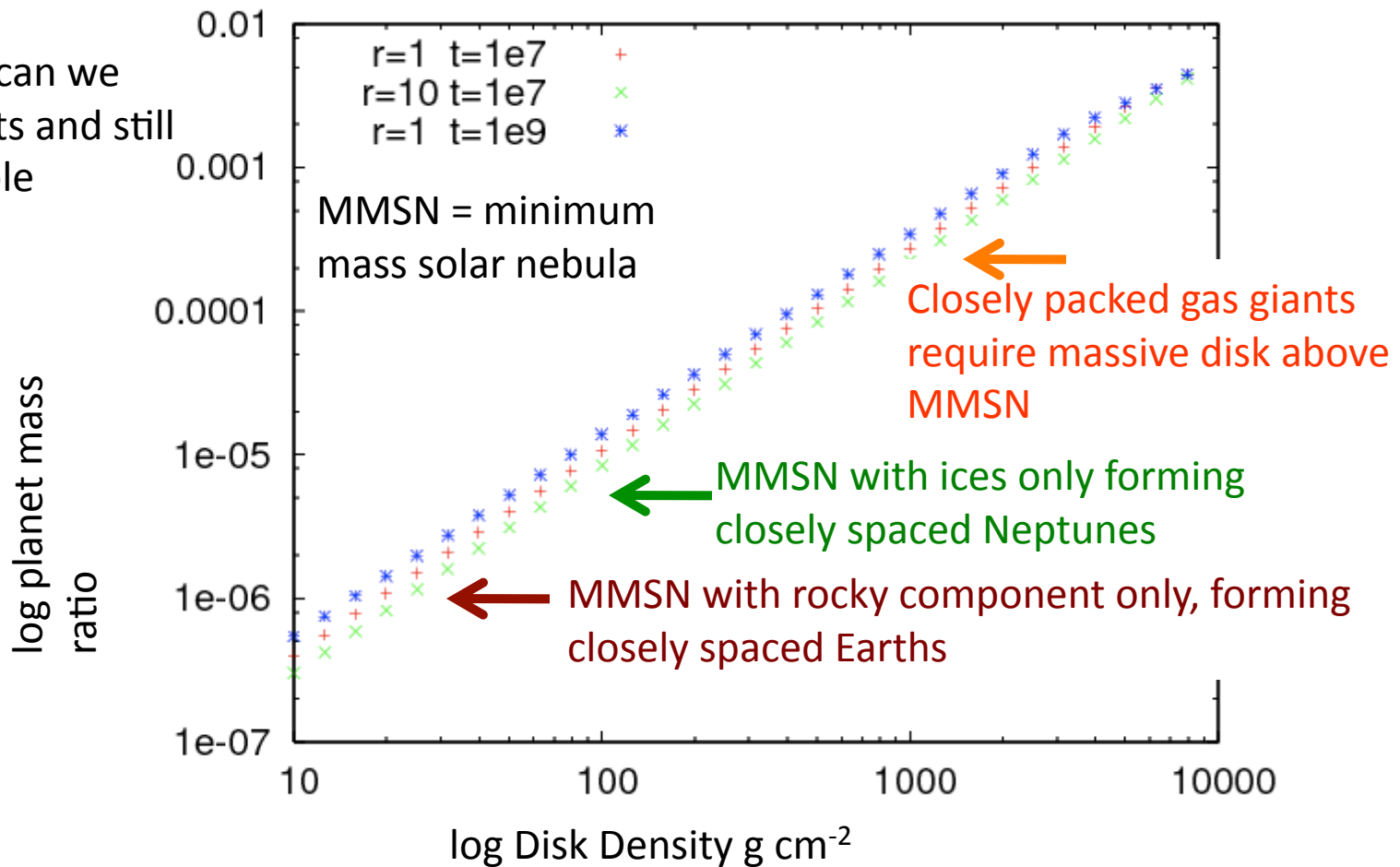
Stable planetary system and unstable planetesimal ones.

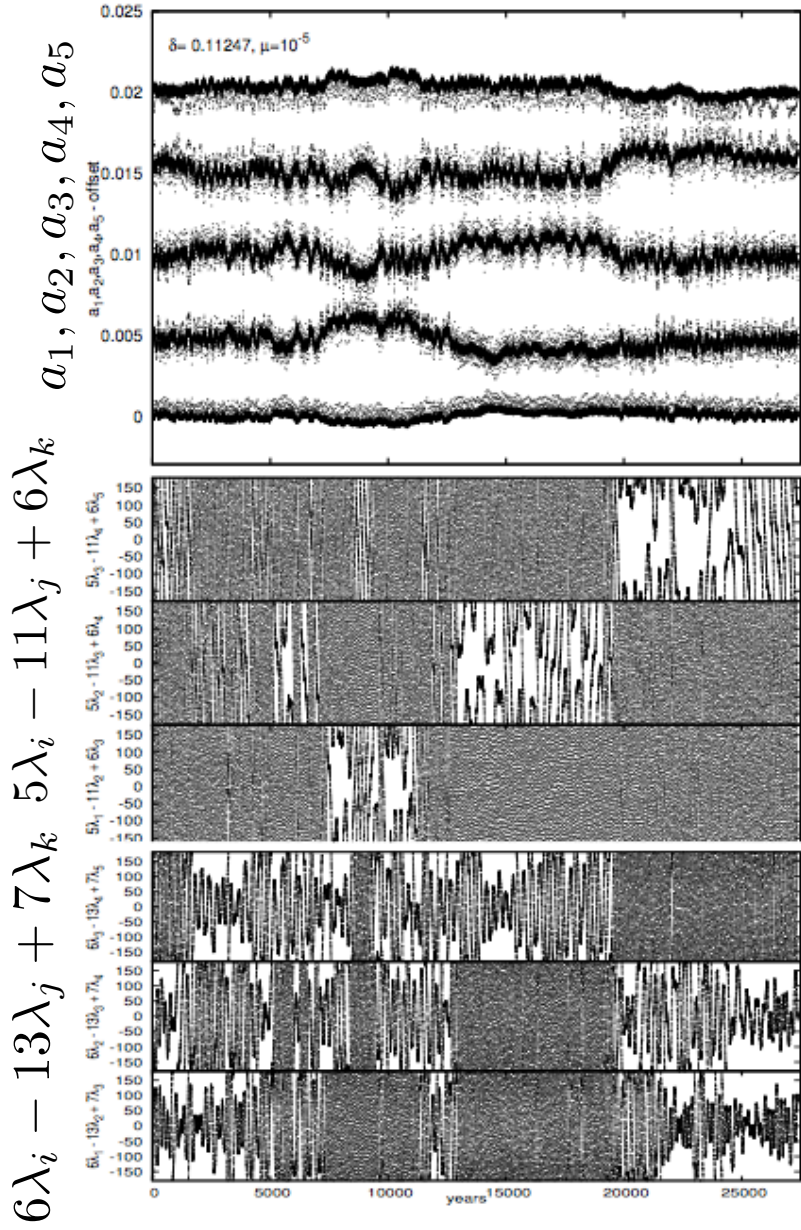
Faber & Quillen 07, Power law for stability based on Chambers et al.96



# Closely packed non-resonant Multiple planet systems

How close can we  
pack planets and still  
have a stable  
system?





# Signatures of Three-body resonances in integrations

Characteristic related motions of three semi-major axes for consecutive bodies.

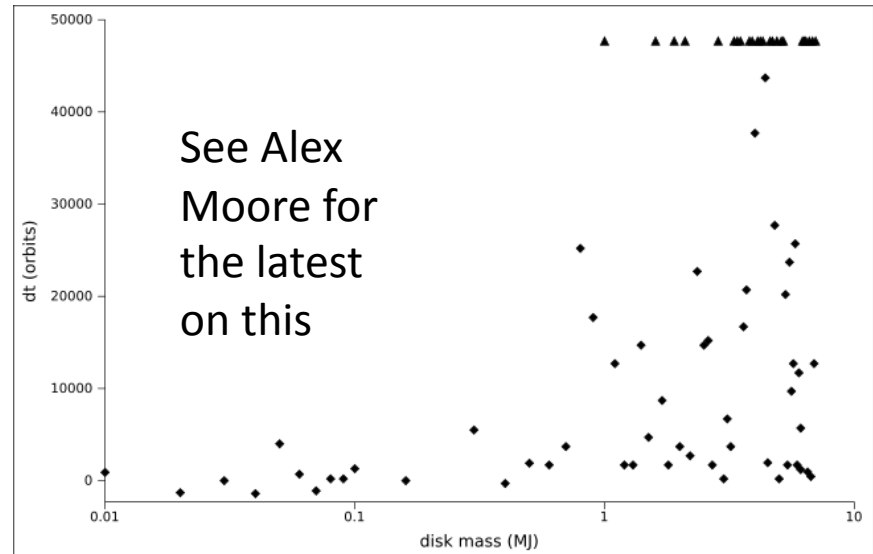
Frozen Laplace angles involving three consecutive bodies.

# Source of InStability in closely packed systems

- Large number of three-body resonances allows continuous wander in closely spaced systems.
- Source of longer term instability
- Resonance overlap criterion

# Islands of stability

- HR8799 system has closely spaced massive planets in presence of debris disk is likely unstable
- 1:2:4 resonance (Goździewski & Migaszewski, Fabricky & Murray-Clay) is a tiny island of stability



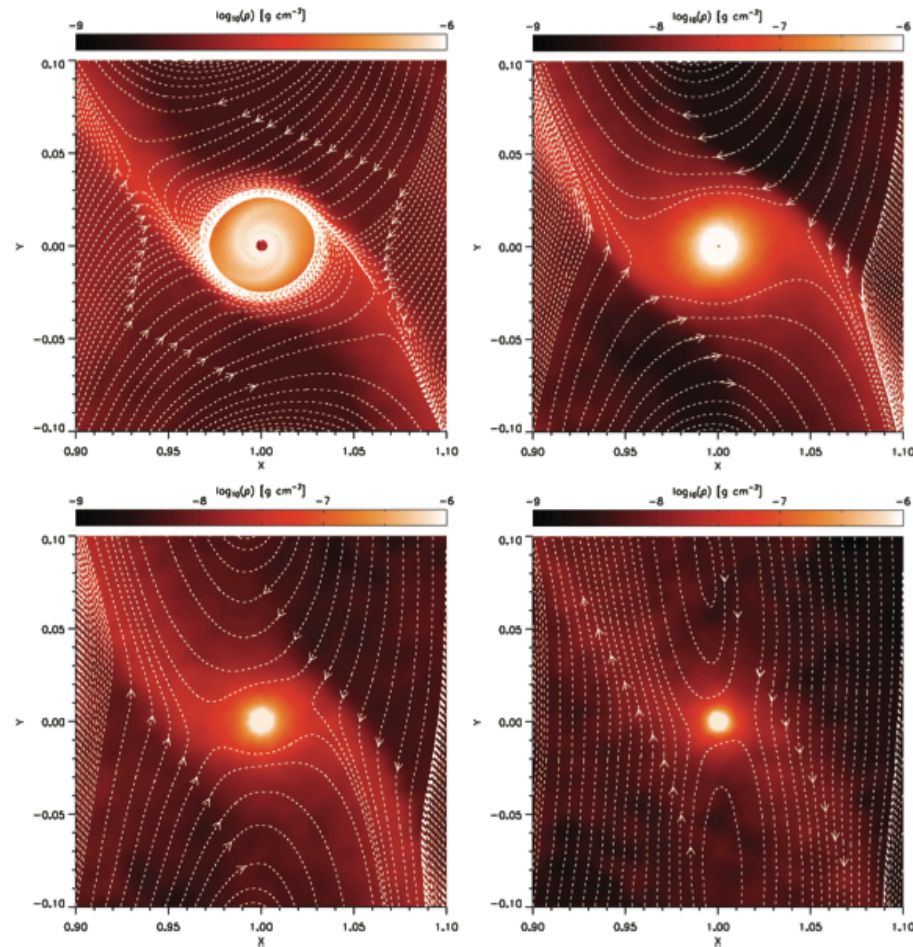
Even small motions in outer planet due to interaction with a disk can vastly decrease or increase time till first planet crossing event

# Circumplanetary disks

Fomalhaut B's unusual color has led to some interesting speculation

- 1) circumJovian accretion disk – e.g., Canup & Ward -- possibly long lived at large radius
- 2) Dust swarm due to irregular satellites (e.g. Bottke et al. 2010, Kennedy & Wyatt 2010)
- 3) Capture of irregular satellites is particularly easy around an outer slowly migrating planet (Hasan et al.)

Ayliffe & Bate 2009



# That was then / this is now

- A few years ago, the proposal that a feature in a disk was caused by a planet was considered a wild possibility. (e.g., will anybody believe it if I propose a Neptune mass object at 119AU?)
- Now multiple planet/disk interactions and scenarios receiving broad spectrum of study