Magnetically Controlled Accretion Flows onto Young Stellar Objects

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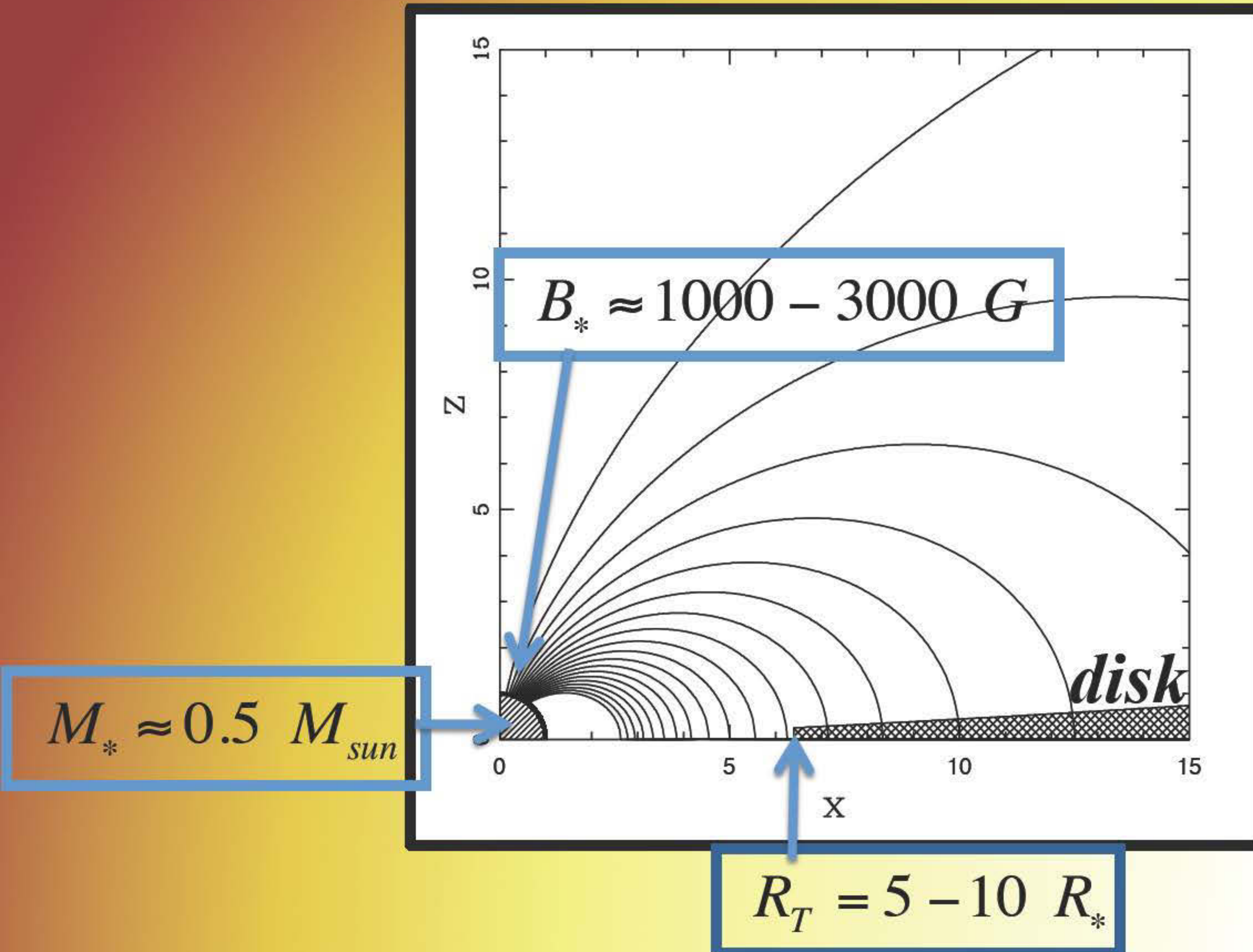
(w: S. Gregory, CalTech)

Motivation

- Most material that ends up in a forming star is processed through the disk; final accretion stage occurs via magnetic field
- T Tauri stars are observed to have magnetic fields with strong Octupole Components (in addition to Dipoles)
- Want to understand the transition through sonic points for magnetically controlled flows in general

The Star/Disk System



Basic Regime of Operation

$$\frac{dM}{dt} \approx 10^{-7} - 10^{-8} \;\; M_{sun} yr^{-1}$$

$$\frac{B^2}{8\pi\rho v^2} \approx 350 - 3000 \;\; (magnetically-controlled)$$

$$\frac{\omega_C}{\Gamma} = \frac{qB}{cmn\sigma v} \approx 10^4 - 10^5 \;\; (well-coupled)$$

$$\frac{B_\perp}{B} = \mathrm{O}\left(8\pi\rho v^2 / B^2\right) < 10^{-3} \;\; (current-free)$$

Equations of Motion

Steady-state flow, polytropic equation of state:

$$\nabla \bullet \left(\rho\ \vec{u}\right) = 0 \qquad\qquad P = K\ \rho^{1+1/n}$$

$$\vec{u} \bullet \nabla \vec{u} + \nabla \Psi + \frac{1}{\rho} \nabla P + \vec{\Omega} \times \left(\vec{\Omega} \times \vec{r}\right) = 0$$

$$\vec{B} = \kappa\ \rho\ \vec{u} \qquad where \qquad \kappa = const$$

Construct Coordinate Systems that follow Magnetic Field Lines

Basis Vectors

covariant:

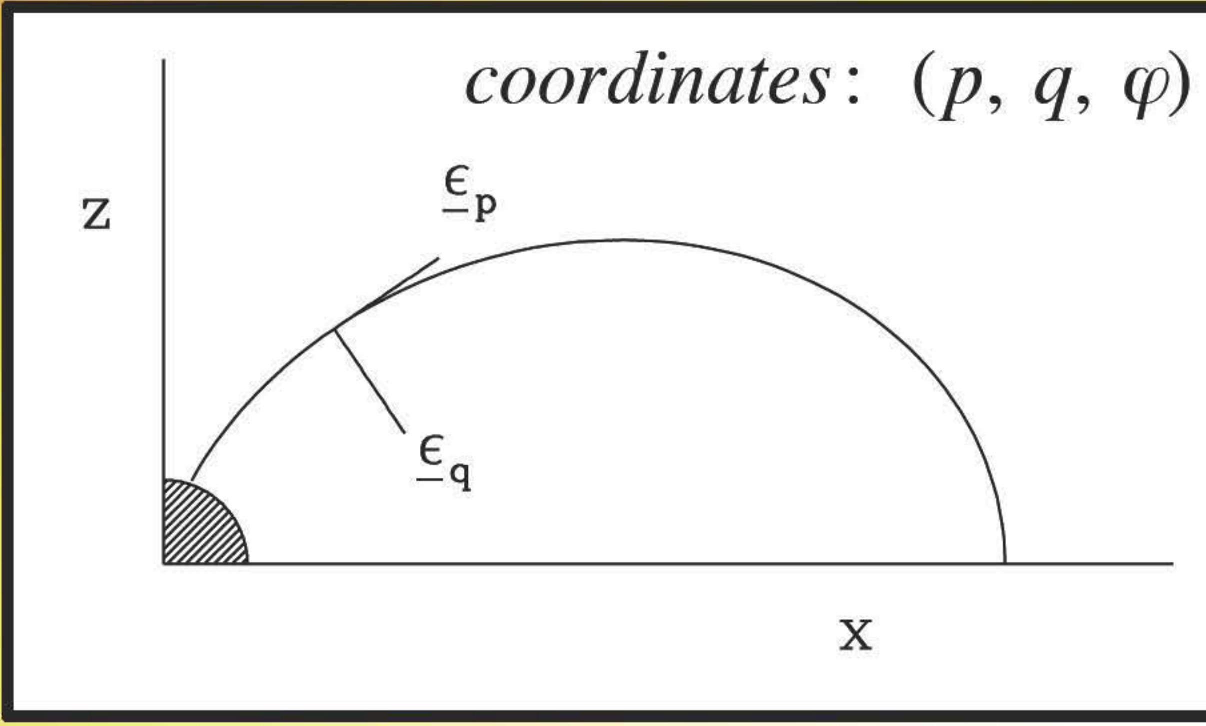
$\underline{\varepsilon}_p = \nabla p$

$\underline{\varepsilon}_q = \nabla q$

contravariant:

$\vec{e}_p = \partial \vec{r} / \partial p$

$\vec{e}_q = \partial \vec{r} / \partial q$



Scale Factors: $h_j = \left| \underline{\varepsilon}_j \right|^{-1}$

Unit Vectors: $\hat{e}_j = \underline{\varepsilon}_p \, h_j = \vec{e}_j \, h_j^{-1}$

Dimensionless Equations of Motion

Steady-state flow along field-line direction:
Fluid fields are functions of coordinate p only.

$$\alpha \frac{\partial u}{\partial p} + u \frac{\partial \alpha}{\partial p} = -\frac{\alpha u}{h_q h_\phi} \frac{\partial}{\partial p}\left(h_q h_\phi\right)$$

$$u \frac{\partial u}{\partial p} + \frac{\alpha^{1/n}}{\alpha} \frac{\partial \alpha}{\partial p} - \omega \xi \sin\theta \left|\nabla p\right|^{-1} \left(\hat{x} \cdot \hat{p}\right) = -\frac{\partial \psi}{\partial p}$$

$$u = \frac{|\vec{u}\,|}{a_S}, \quad \alpha = \frac{\rho}{\rho_1}, \quad \psi = \frac{\Psi}{a_S^2}, \quad b \equiv \frac{GM_P}{R_* a_S^2} \quad \omega \equiv \left(\frac{\Omega R_*}{a_S}\right)^2$$

Integrated Equations of Motion

$$\alpha \ u \ h_q h_\phi = \lambda$$

$$\frac{1}{2} u^2 + n\alpha^{1/n} + \psi - \omega \ I = \varepsilon$$

$$I = \int \ \xi \sin\theta \ |\nabla p|^{-1} (\hat{x} \cdot \ \hat{p}) \ dp$$

$$\lambda = mass \ \ accretion \ \ rate$$

$$\varepsilon = energy$$

Sonic Transition Condition

$$\alpha^{1/n} \frac{\Upsilon}{\xi} + \omega \Lambda = \frac{b}{\xi^2}$$

$$\Upsilon(\xi,\theta) \equiv \frac{\xi}{h_p} \frac{\partial h_p}{\partial \xi} \quad (where \quad h_p = h_q h_\phi)$$

$$\Lambda(\xi,\theta) \equiv \xi \sin\theta \left(\frac{\partial p}{\partial \xi} \right)^{-1} \left(\hat{x} \cdot \nabla p \right)$$

General Constraint on Steady Polytropic Transonic Accretion Flow

Observations show that flow must be ***transonic***. ***Steady-state*** accretion solutions that pass through the sonic point and approach free-fall speed near the star must satisfy the constraint:

$$n > \ell + 3/2 \rightarrow 9/2 \ (\textit{octupole})$$

Steady flow must be nearly isothermal for fields with higher order multipoles.

Score Card

b *and* ω : *system parameters* $(n \to \infty)$

λ *and* ε : *conserved quantities*

$\Upsilon(\xi,\theta)$ *and* $\Lambda(\xi,\theta)$: *functions that specify magnetic field geometry*

field lines $\Rightarrow q = const \Rightarrow \theta = F(\xi)$

Dipole Coordinate System

$$\vec{B} = B_0\left[\xi^{-3}\left(3\cos\theta\ \hat{r} - \hat{z}\right)\right] \quad where \quad \xi = r/R_*$$

$$p = -\xi^{-2}\cos\theta \quad and \quad q = \xi^{-1}\sin^2\theta$$

$$\left.\begin{aligned} \nabla p &= 2\xi^{-3}\cos\theta\ \hat{r} + \xi^{-3}\sin\theta\ \hat{\theta} \\ \nabla q &= -\xi^{-2}\sin^2\theta\ \hat{r} + 2\xi^{-2}\cos\theta\sin\theta\ \hat{\theta} \end{aligned}\right\} \quad \nabla p \bullet \nabla q = 0$$

$$h_p = \xi^3\left[4\cos^2\theta + \sin^2\theta\right]^{-1/2}$$

$$h_p = \frac{\xi^2}{\sin\theta}\left[4\cos^2\theta + \sin^2\theta\right]^{-1/2} \qquad \left(h_\phi = \xi\sin\theta\right)$$

Ancillary Functions

For Coordinate System that follows Dipole Magnetic Field Lines:

$$\Upsilon = \Upsilon(\xi) = \frac{3}{2}\,\frac{8 - 5q\xi}{4 - 3q\xi}$$

$$\Lambda = \Lambda(\xi) = \frac{3}{2} q\,\xi^2$$

along field line labeled by the coordinate q

Solutions (Isothermal Limit)

Accretion flow follows magnetic field lines, which are lines of constant coordinate q.

$$3\xi\frac{8-5q\xi}{4-3q\xi}=2b-3\ \omega\ q\ \xi^4$$

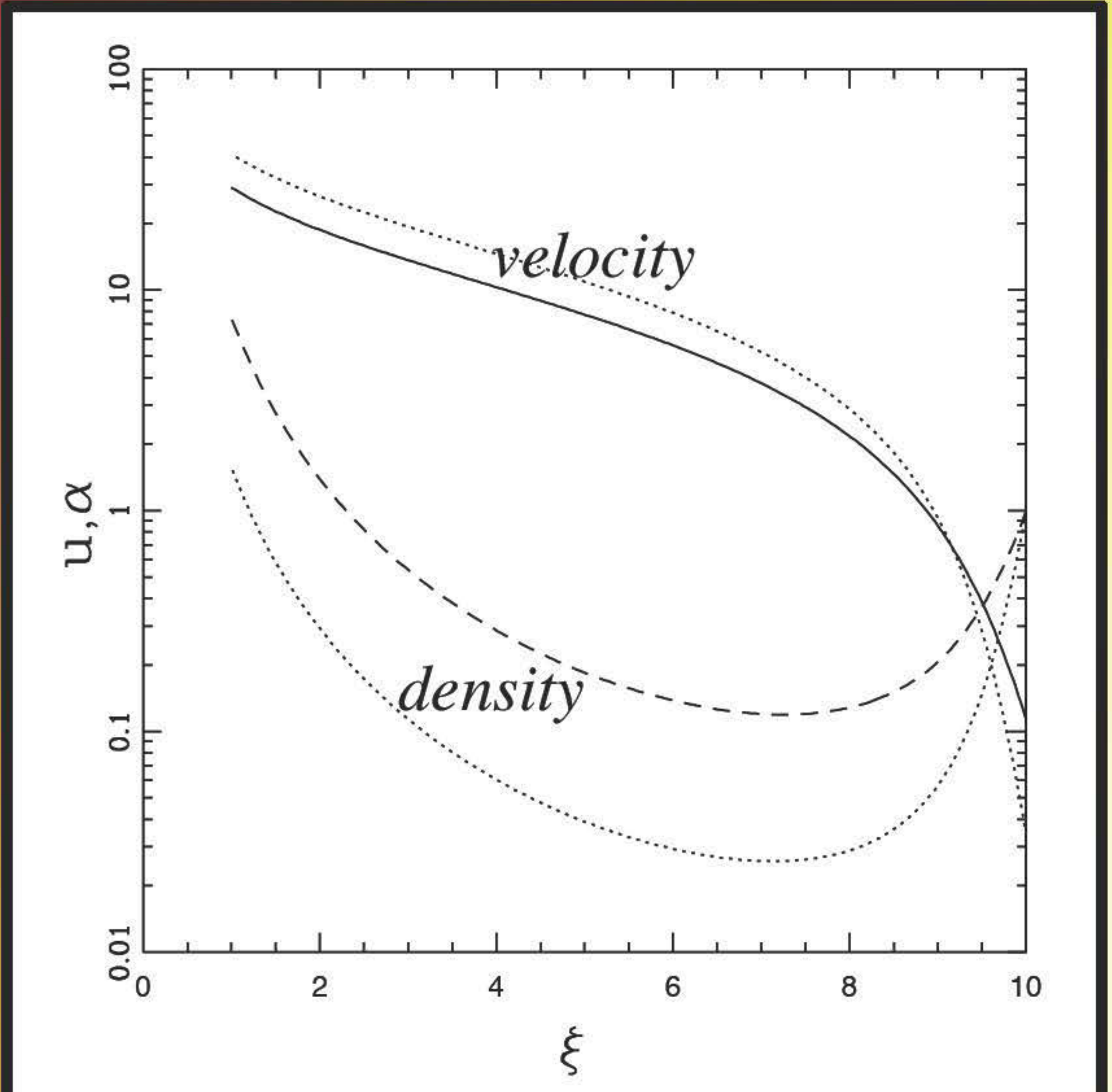
(Sonic Point Condition)

$$\log\lambda-\frac{1}{2}\lambda^2=3\log\xi_s-\frac{1}{2}\log(4-3\xi_s)-\frac{1}{2}$$

$$+b\left(\frac{1}{\xi_s}+\frac{1}{2}\xi_s^2-\frac{3}{2}\right)$$

(Mass Accretion Rate)

Dipole Accretion Solution



dipole fields

isothermal flow

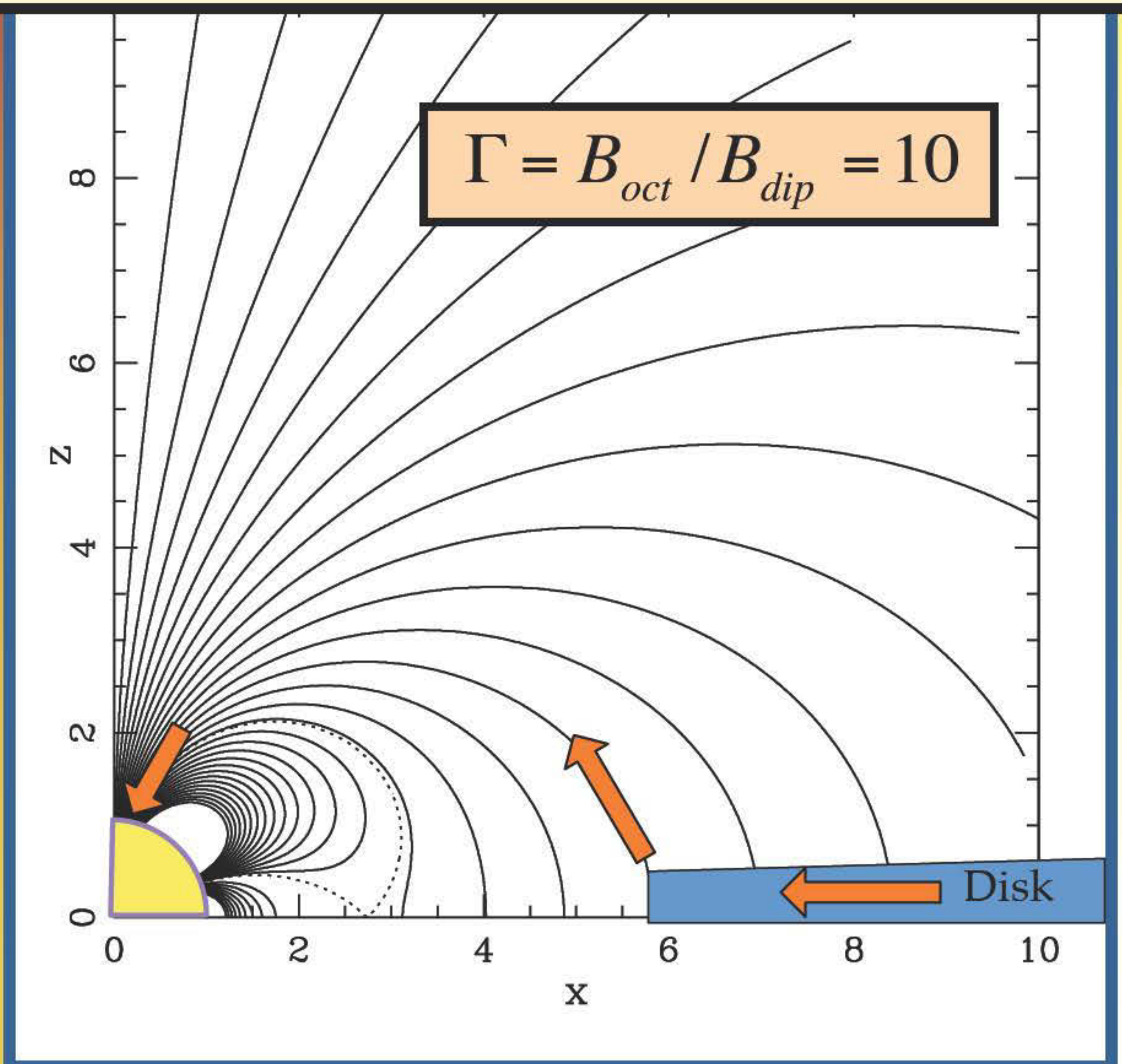
$\xi_d = 10$

$\xi_* = 1$

$b = 500$

$b = 1000$ (*dots*)

Dipole + Octupole Configuration



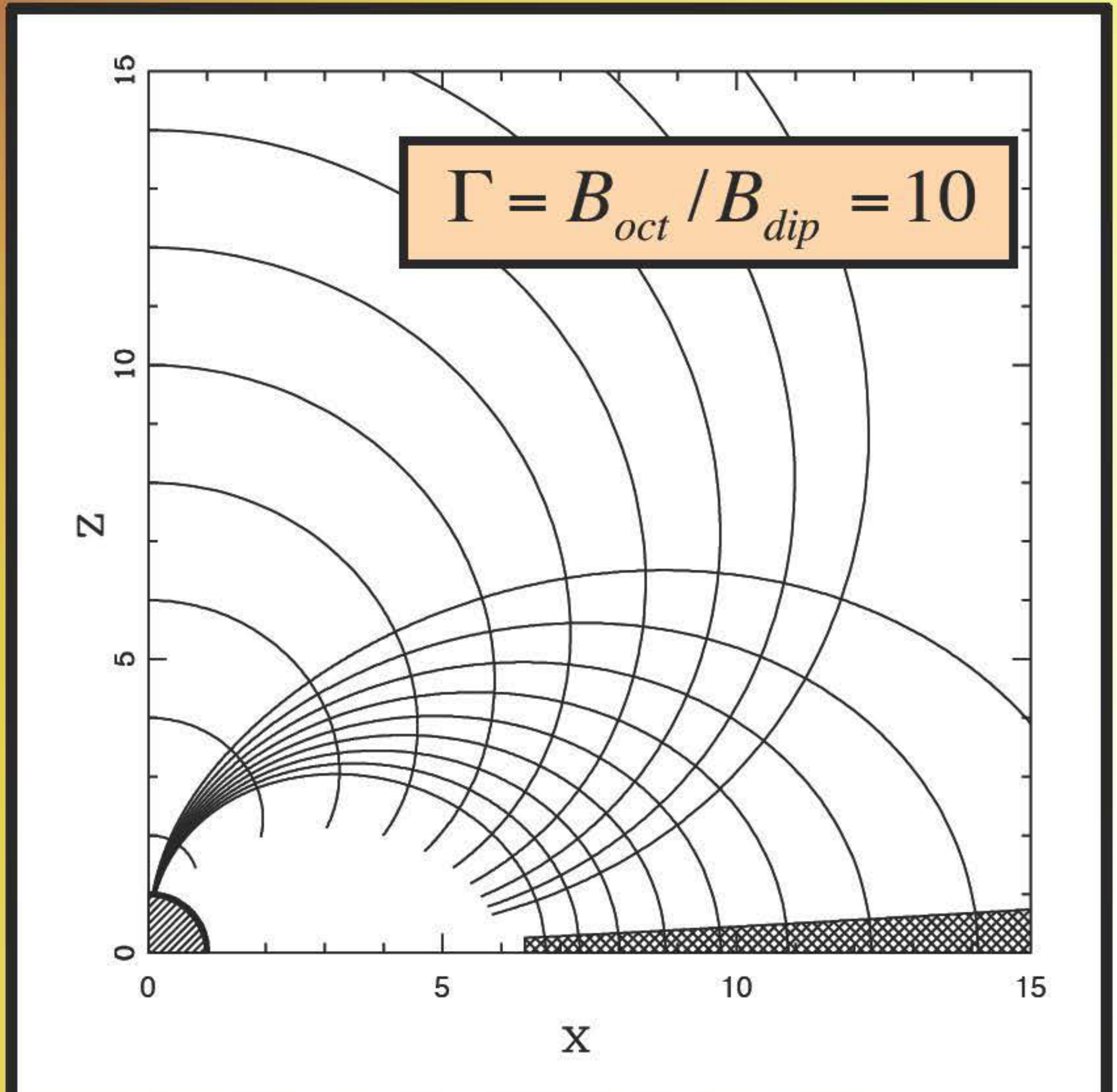
Dipole + Octupole Coordinate System

$$\vec{B} = B_{dip}\xi^{-3}\left(2\cos\theta\ \hat{r} + \sin\theta\ \hat{\theta}\right)\ +$$

$$\frac{1}{2}B_{oct}\xi^{-5}\left[\left(5\cos^2\theta - 3\right)\cos\theta\ \hat{r} + \frac{3}{4}\left(5\cos^2\theta - 1\right)\sin\theta\ \hat{\theta}\right]$$

$$\left.\begin{aligned} p &= -\frac{\Gamma}{4}\xi^{-4}\left(5\cos^2\theta - 3\right)\cos\theta - \xi^{-2}\cos\theta \\ q &= \frac{\Gamma}{4}\xi^{-3}\left(5\cos^2\theta - 1\right)\sin^2\theta + \xi^{-1}\sin^2\theta \end{aligned}\right\}\ \textit{where}\quad \Gamma \equiv \frac{B_{oct}}{B_{dip}}$$

Dipole + Octupole Coordinate System



Dipole + Octupole Scale Factors

$$h_p = \xi^5 \; [f^2 \cos^2\theta + g^2 \sin^2\theta]^{-1/2}$$

$$h_q = \xi^4 \; (\sin\theta)^{-1} \; [f^2 \cos^2\theta + g^2 \sin^2\theta]^{-1/2}$$

$$\textit{where} \quad f = \Gamma\,(5\cos^2\theta - 3) + 2\xi^2$$

$$\textit{and} \quad g = (3/4)\Gamma\,(5\cos^2\theta - 1) + \xi^2$$

$$\sin^2\theta = \frac{2}{5\Gamma}\left\{(\xi^2 + \Gamma) - \left[(\xi^2 + \Gamma)^2 - 5\Gamma q \xi^3\right]^{1/2}\right\}$$

$$\textit{so that} \quad f,\; g,\; h_p,\; h_q = \textit{Function}(\xi \textit{ only})$$

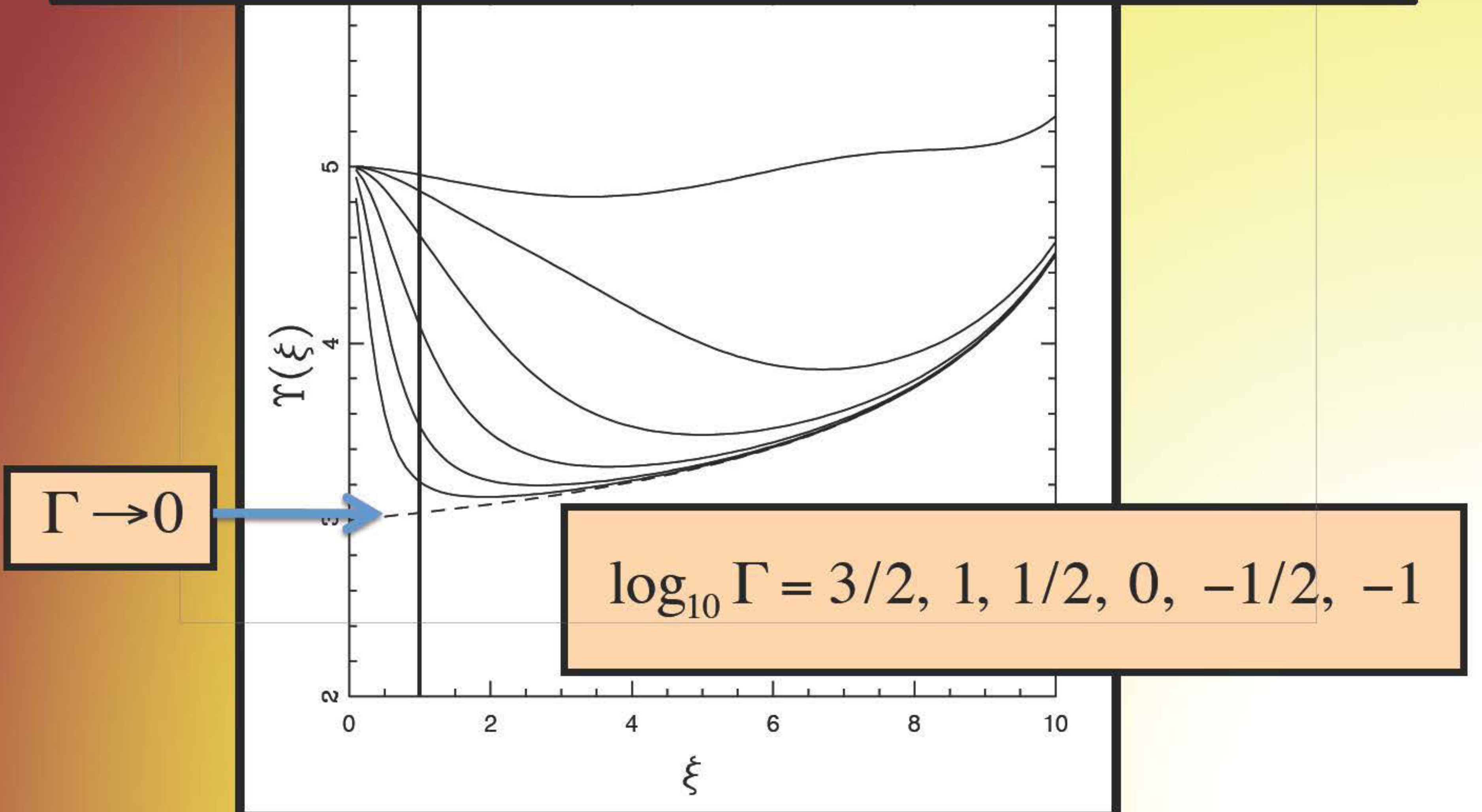
Ancillary Functions for Dipole + Octupole Configuration

$$\Lambda(\xi) = \frac{3\xi}{5\Gamma}\left\{1 + \frac{g(\xi)}{f(\xi)}\right\}\left\{(\xi^2 + \Gamma) - \left[(\xi^2 + \Gamma)^2 - 5\Gamma q \xi^3\right]^{1/2}\right\}$$

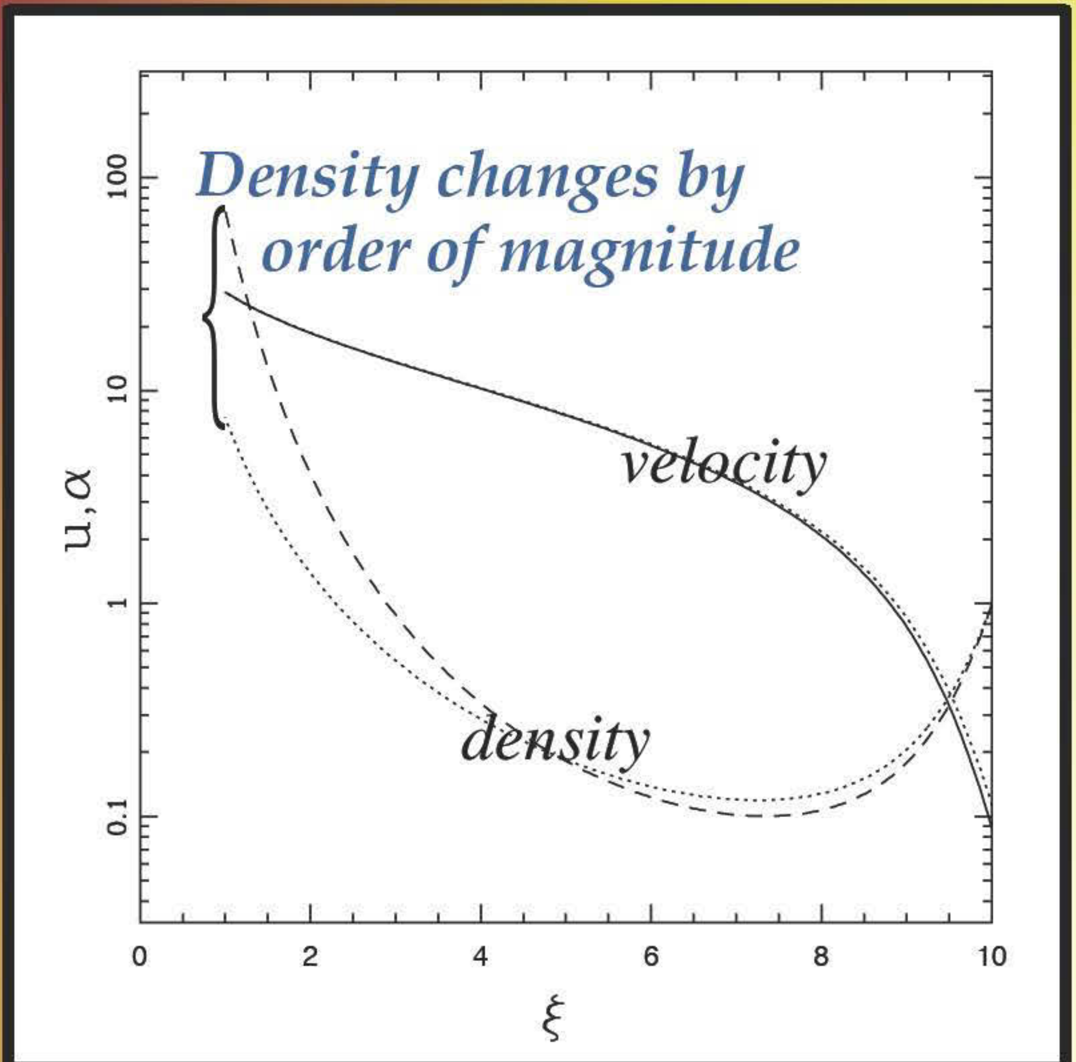
$$\Upsilon(\xi) = 5 - \left[f^2 + (g^2 - f^2)\frac{1}{5\Gamma}(2\xi^2 + 2\Gamma - f)\right]^{-1}\frac{\xi}{5\Gamma} \times$$

$$\left\{5\Gamma f\, f_\xi + \left[g\left(\frac{3f_\xi}{4} - \xi\right) - f\, f_\xi\right](2\xi^2 + 2\Gamma - f) + (g^2 - f^2)\left(2\xi - \frac{f_\xi}{2}\right)\right\}$$

Index of Divergence Operator (Dipole + Octupole)



Accretion Solution (Dip+Oct)



isothermal flow

octupole $\Gamma = 10$

$\xi_d = 10$

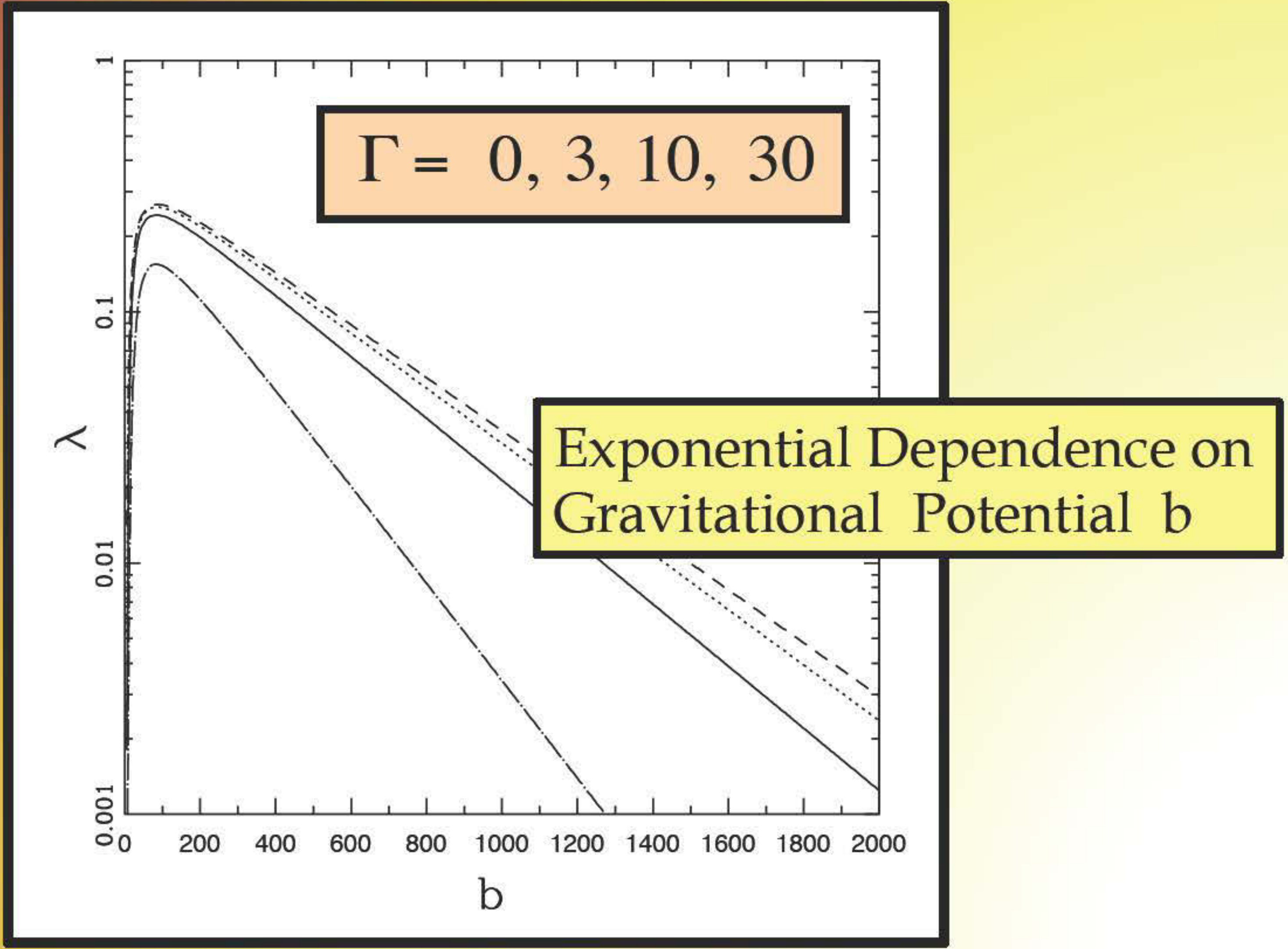
$\xi_* = 1$

$b = 500$

dots = dipole solution

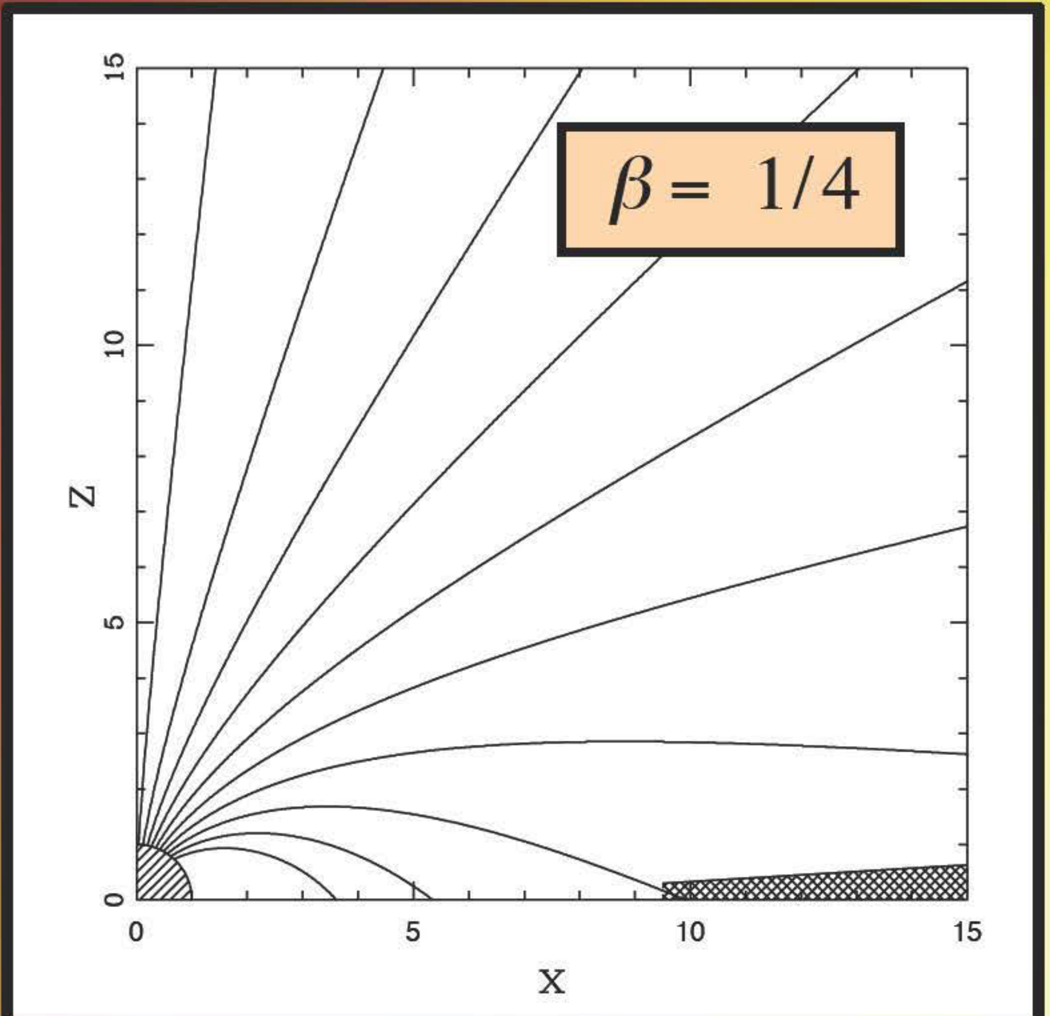
dashes = full solution

Dimensionless Accretion Rate



To a man with a hammer, everything looks like a nail

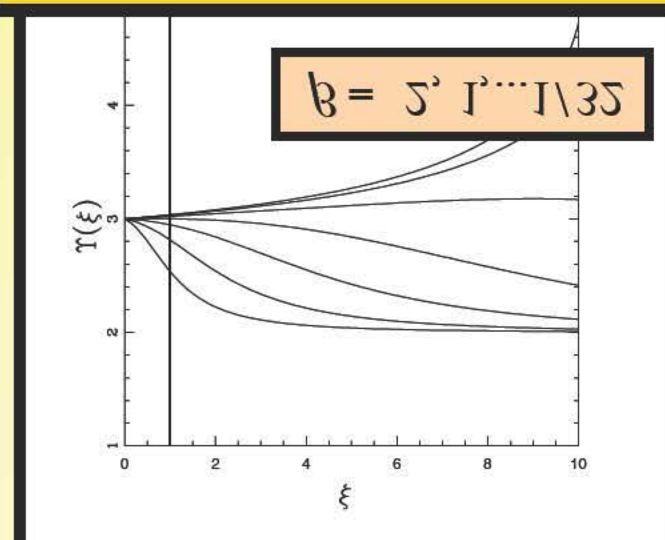
Dipole + Split-Monopole



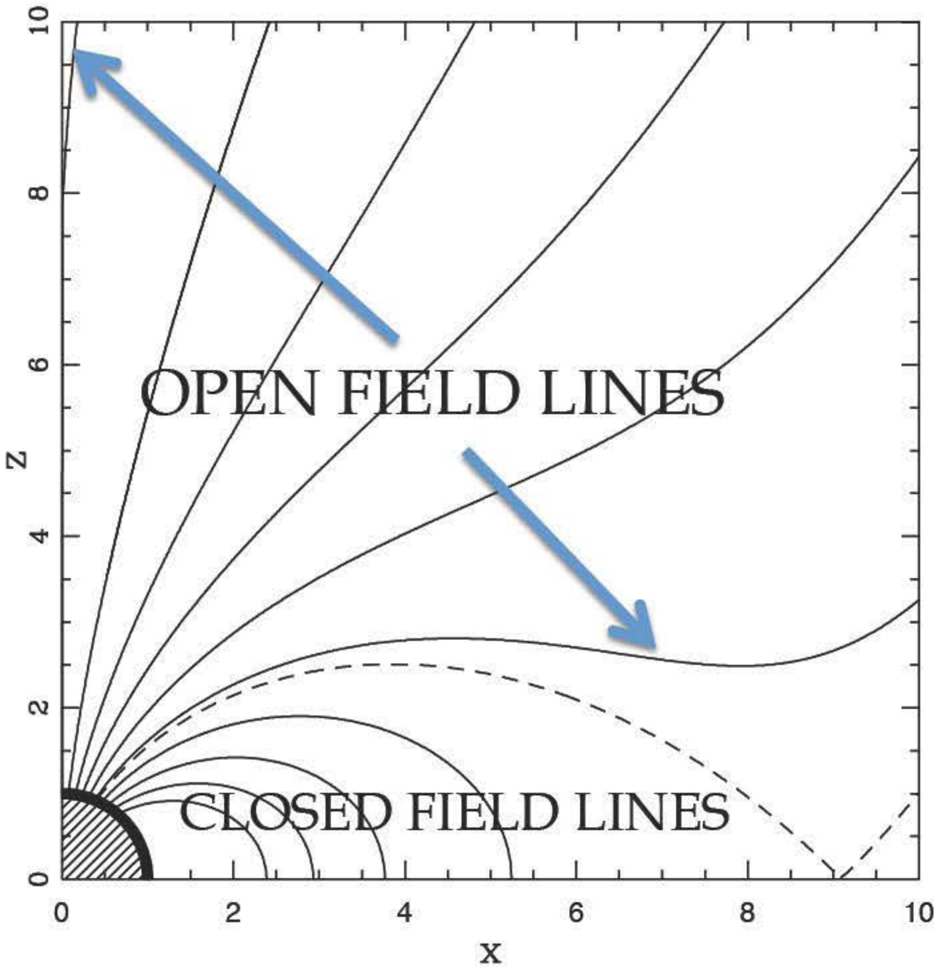
$$p = -\xi^{-2}\cos\theta - \beta\xi^{-1}$$

$$q = \xi^{-1}\sin^2\theta - \beta\cos\theta$$

$$where \quad \beta \equiv 2B_{rad}/B_{dip}$$



Planet Dipole in Stellar z-Field



$$p = \left(\beta\xi - \xi^{-2}\right)\cos\theta$$

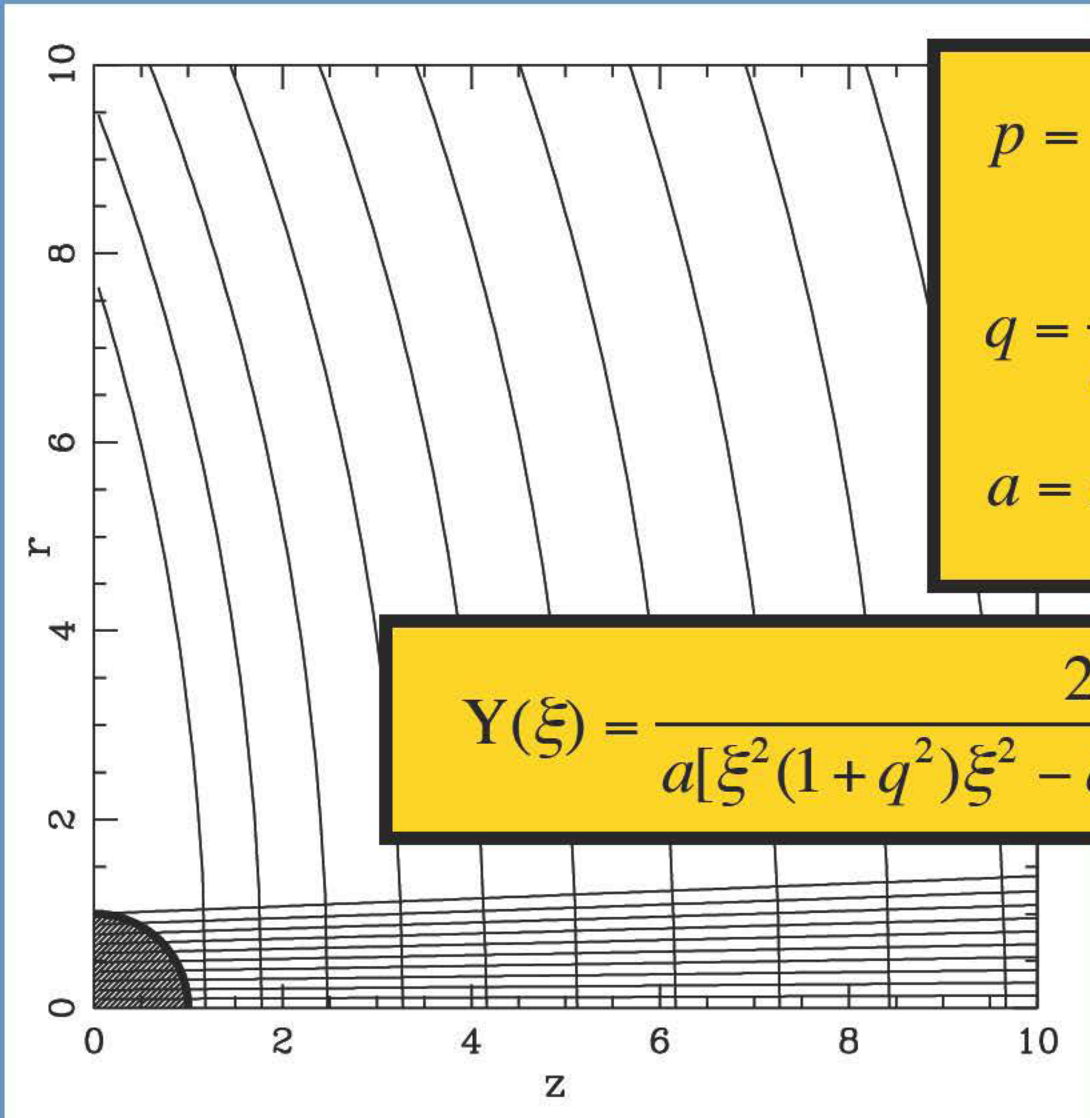
$$q = \left(\beta\xi^{2} + 2/\xi\right)^{1/2}\sin\theta$$

$$\beta = (B_{*}R_{*}^{3}/\varpi^{3})/B_{P}$$

Field lines are open near planetary pole and are closed near the equator. Fraction of open field lines:

$$F_{open} = 1 - \left[1 - \frac{3\beta^{1/3}}{2+\beta}\right]^{1/2}$$

Planet in Stellar Split-Monopole Field



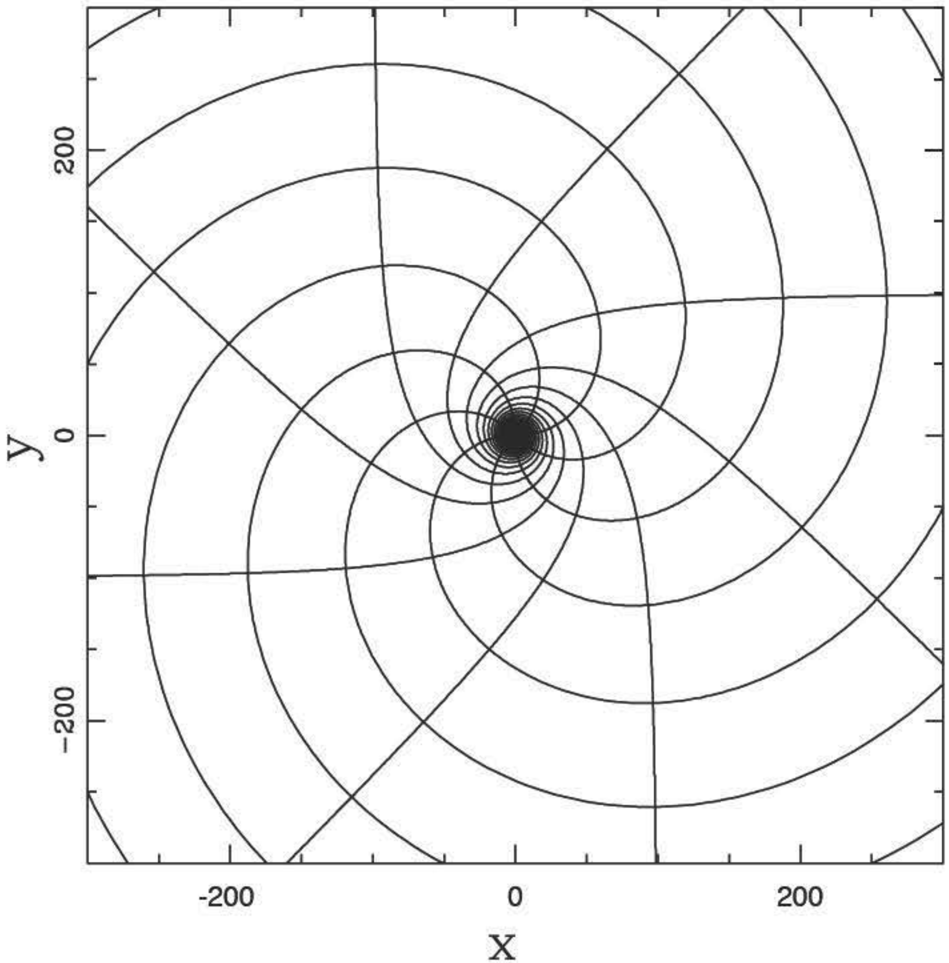
$$p = -\left(a^2 + \xi^2 + 2a\xi\cos\theta\right)^{-1/2}$$

$$q = \frac{\xi\sin\theta}{a + \xi\cos\theta}$$

$$a = radius\ of\ orbit$$

$$\mathrm{Y}(\xi) = \frac{2\xi^2(1+q^2)}{a[\xi^2(1+q^2)\xi^2 - a^2q^2]^{1/2} + [(1+q^2) - a^2q^2]}$$

Parker Spiral in Equatorial Plane



$$p = A\log\left(1-\frac{1}{\xi}\right) + \phi$$

$$q = \xi - 1 - \log\xi - A\phi$$

$$A \equiv V_{wind}/(\omega R_*) \approx 100$$

(A *sets shape of spiral*)

Summary

- Construction of coordinate systems (p,q)
- Generalizes to many astrophysical problems
- Can find sonic points and dimensionless mass accretion rates analytically
- Dipole + Octupole system: flow density (10x)larger, hot spot has higher temperature
- Magnetic truncation radius changes
- General constraint on steady transonic flow:

$$n > \ell + 3/2 \Rightarrow \textit{nearly} \quad \textit{isothermal}$$

(Adams & Gregory, 2012, ApJ, 744, 55; Adams, 2011, ApJ, 730, 27)

Dragons

- *This use of coordinate systems in this context only works for potential fields (no currents)*
- *The formalism has been developed for two-dimensional systems; can work for three-dimensional systems in principal, but complicated in practice*
- *Treatment (thus far) limited to steady-state (time independent) magnetic fields: magnetostatics not MHD*